CODE:2401- AG-4-TS-XII-20-21 पजियन क्रमांक REG.NO:-TMC -D/79/89/36

General Instructions :-

(i) **All** Question are compulsory :

(ii) This question paper contains **36** questions.

(iii) Question **1-20** in **PART- A are** Objective type question carrying **1** mark each.

(iv) Question **21-26** in **PART -B** are sort-answer type question carrying **2** mark each.

(v) Question **27-32** in **PART -C** are long-answer-**I** type question carrying **4** mark each.

(vi) Question **33-36** in **PART -D** are long-answer-**II** type question carrying **6** mark each

(vii) You have to attempt only one if the alternatives in all such questions.

(viii) Use of calculator is not permitted.

(ix) Please check that this question paper contains 8 printed pages.

Code number given on the right-hand side of the question paper should be written on the title page of the answer-book by the candidate.

EXAMINATION 2020 -21

Time : 3 Hours Maximum Marks : 80

CLASS – XII **MATHEMATICS**

PART – A (Question 1 to 20 carry 1 mark each.)

SECTION I: Single correct answer type

This section contains 12 multiple choice question. Each question has four choices (A) , (B) , (C) &(D) out of which ONLY ONE is correct .

Q.1 $A=\begin{bmatrix}4 & 6 & -1\\3 & 0 & 2\\1 & -2 & 5\end{bmatrix}, B=\begin{bmatrix}2 & 4\\0 & 1\\-1 & 2\end{bmatrix}, C=\begin{bmatrix}3\\1\\2\end{bmatrix}$, then the expression which is not defined is

(a) $A^2+2B-2A$ (b) CC' (c) $B'C$ (d) AB (a) By inspection, A^2 and A matrix is of order 3×3, while B matrix is of order 3×2. Therefore, $A^2+2B-2A$ is not defined.

Q.2 If $\mathbf{r}.\mathbf{i}=\mathbf{r}.\mathbf{j}=\mathbf{r}.\mathbf{k}$ and $|\mathbf{r}|=3$, then $\mathbf{r}=$

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	(a) $\pm 3(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (b) $\pm\frac{1}{3}(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (c) $\pm\frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (d) $\pm\sqrt{3}(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (d) Let $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$. Since $\mathbf{r}.\mathbf{i}=\mathbf{r}.\mathbf{j}=\mathbf{r}.\mathbf{k} \Rightarrow x=y=z$ .....(i) Also $\lvert\mathbf{r}\rvert=\sqrt{x^2+y^2+z^2}=3 \Rightarrow x=\pm\sqrt{3}$, {By (i)} Hence the required vector $\mathbf{r}=\pm\sqrt{3}(\mathbf{i}+\mathbf{j}+\mathbf{k})$. **Trick :** As the vector $\pm\sqrt{3}(\mathbf{i}+\mathbf{j}+\mathbf{k})$ satisfies both the conditions
Q.3	From a pack of 52 cards two are drawn with replacement. The probability, that the first is a diamond and the second is a king, is (a) $\frac{1}{26}$ (b) $\frac{17}{2704}$ (c) $\frac{1}{52}$ (d) None of these (c) Required probability $=P(\text{Diamond}).P(\text{king})=\frac{13}{52}.\frac{4}{52}=\frac{1}{52}$.
Q.4	The distance of the point (4, 3, 5) from the y-axis is (a) $\sqrt{34}$ (b)5 (c) $\sqrt{41}$ (d) $\sqrt{15}$ (c)Distance from y-axis is $\sqrt{x^2+z^2}$ $=\sqrt{4^2+5^2}=\sqrt{16+25}=\sqrt{41}$.
Q.5	If $4\sin^{-1}x+\cos^{-1}x=\pi$, then x is equal to (a) 0 (b) $\frac{1}{2}$ (c) $-\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$ (b) We know that $4\sin^{-1}x+\cos^{-1}x=\pi$ $\Rightarrow 3\sin^{-1}x+\sin^{-1}x+\cos^{-1}x=\pi$ $\Rightarrow 3\sin^{-1}x=\pi-\frac{\pi}{2}=\frac{\pi}{2}$ $\Rightarrow \sin^{-1}x=\pi/6 \Rightarrow x=\sin\frac{\pi}{6}=\frac{1}{2}$.
Q.6	The probability of hitting a target by three marksmen are $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that one and only one of them will hit the target when they fire simultaneously, is (a) $\frac{11}{24}$ (b) $\frac{1}{12}$ (c) $\frac{1}{8}$ (d) None of these (a) Here $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(C)=\frac{1}{4}$ $\therefore$Hence required probability $=P(A)P(\overline{B})P(\overline{C})+P(\overline{A})P(B)P(\overline{C})+P(\overline{A})P(\overline{B})P(C)$.

Q.7	$\int \frac{x\,dx}{1-x\cot x} =$ (a) $\log(\cos x - x\sin x) + c$ (b) $\log(x\sin x - \cos x) + c$ (c) $\log(\sin x - x\cos x) + c$ (d) None of these (c) $\int \frac{x\,dx}{1-x\cot x} = \int \frac{x\,dx}{1-x\frac{\cos x}{\sin x}} = \int \frac{x\sin x}{\sin x - x\cos x}dx$ $= \int \frac{dt}{t} = \log t = \log(\sin x - x\cos x) + c.$ {Putting $\sin x - x\cos x = t$, $\Rightarrow [\cos x - (-x\sin x + \cos x)]\,dx = dt \Rightarrow x\sin x\,dx = dt$}
Q.8	The necessary condition for third quadrant region in xy-plane, is (a) $x > 0, y < 0$ (b) $x < 0, y < 0$ (c) $x < 0, y > 0$ (d) $x < 0, y = 0$ ans B
Q.9	The perpendicular distance of the point (2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ is (a)3 (b) 5 (c) 7 (d) 9 (c) The perpendicular distance of (2, 4, – 1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ is $\left\{(2+5)^2 + (4+3)^2 + (-1-6) - \left[\frac{1(2+5)+4(4+3)-9(-1-6)}{\sqrt{1+16+81}}\right]^2\right\}^{1/2}$ $= \sqrt{147 - \left(\frac{98}{\sqrt{98}}\right)^2} = \sqrt{147-98} = \sqrt{49} = 7$.
Q.10	If $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \\ \frac{\sqrt{x}}{\sqrt{(16+\sqrt{x})}-4}, & \text{when } x > 0 \end{cases}$, is continuous at $x = 0$, then the value of 'a' will be (a) 8 (b) – 8 (c) 4 (d) None of these (a) $\lim_{x\to 0-} f(x) = \lim_{x\to 0-}\left(\frac{2\sin^2 2x}{(2x)^2}\right)4 = 8$ $\lim_{x\to 0+} f(x) = \lim_{x\to 0+}\sqrt{16+\sqrt{x}}+4 = 8$. Hence $a = 8$.
	Fill in the blanks (Q11 – Q15)
Q.11	If A and B are invertible matrices of order 3, $\|A\| = 2$ and $\|(AB)^{-1}\| = -\frac{1}{6}$,

	then $\|B\|$ …………... $\frac{1}{\|AB\|}=-\frac{1}{6}\Rightarrow\frac{1}{\|A\|\|B\|}=-\frac{1}{6}\Rightarrow\|B\|=-3.$
Q.12	If $y=\log_{10}x+\log_x 10+\log_x x+\log_{10}10$, then $\frac{dy}{dx}=-----$ $\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)^2}$
Q.13	If $\begin{bmatrix}1 & -1 & x\end{bmatrix}\begin{bmatrix}0 & 1 & -1\\ 2 & 1 & 3\\ 1 & 1 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}=0$, Then $x=--------$. Ans x =2.
Q.14	The tangent to the curve $y=ax^2+bx$ at $(2,-8)$ is parallel to x-axis. Then a = …….& b = ……..$a=2$ $b=-8$
Q.15	If $\vec{a}$ and $\vec{b}$ are two non-collinear unit vectors such that $\|\vec{a}+\vec{b}\|=\sqrt{3}$, Then $(2\vec{a}-5\vec{b}).(3\vec{a}+\vec{b})=-----$ Ans: $-\frac{11}{2}$ **OR** For two non zero vector $\vec{a}$ and $\vec{b}$ write when $\|\vec{a}+\vec{b}\|=\|\vec{a}\|+\|\vec{b}\|$ holds if ------------ Ans : they are like parallel vector
	(Q16 - Q20) Answer the following questions
Q.16	Evaluate: $\int_0^{\pi/2}\frac{\cos^2 x dx}{1+3\sin^2 x}$. Let $I=\int_0^{\pi/2}\frac{\cos^2 x dx}{1+3\sin^2 x}$ $\Rightarrow I=\int_0^{\pi/2}\frac{\sec^2 x dx}{\sec^4 x+3\sec^2 x\tan^2 x}$ $\therefore I=\int_0^{\pi/2}\frac{\sec^2 x dx}{\sec^2 x(1+\tan^2 x+3\tan^2 x)}$ $I=\int_0^{\pi/2}\frac{\sec^2 x dx}{(1+\tan^2 x)(1+4\tan^2 x)}$ $I=\int_0^{\infty}\frac{dt}{(1+t^2)(1+4t^2)}$ $\Rightarrow I=-\frac{1}{3}\int_0^{\infty}\left(\frac{1}{1+t^2}-\frac{4}{1+4t^2}\right)dt$ $=\frac{\pi}{6}$.
Q.17	Evaluate: $\int\frac{dx}{\sqrt{\sin^3 x\sin(x+\alpha)}}$. **Ans** $-2\operatorname{cosec}\alpha\sqrt{(\cos\alpha+\cot x\sin\alpha)}+c$
Q.17	Evaluate: $\int\frac{(\sin x-x\cos x)dx}{x(x+\sin x)}$

	$\int \frac{\sin x - x\cos x}{x(x+\sin x)}dx \quad I = \int \frac{x+\sin x - x - x\cos x}{x(x+\sin x)}dx$ $I = \int\left(\frac{x+\sin x}{x(x+\sin x)} - \frac{x + x\cos x}{x(x+\sin x)}\right)dx \qquad I = \int \frac{1}{x}dx - \int \frac{1+\cos x}{x+\sin x}dx$ $I = \log\lvert x\rvert - \log\lvert t\rvert + C$ **OR** Evaluate: $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\, dx$ **Solution:** $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}}\, dx = \int \sqrt{\frac{\cos x}{1-\cos^3 x}} \sin x\, dx$ $= -\int \sqrt{\frac{t}{1-t^3}}\, dt = -\int \frac{\sqrt{t}}{\sqrt{1-(t^{3/2})^2}}\, dt = -\frac{2}{3}\int \frac{\frac{3}{2}\sqrt{t}}{\sqrt{1-(t^{3/2})^2}}\, dt$ $= \frac{2}{3}\cos^{-1}(t^{3/2}) + c = \frac{2}{3}\cos^{-1}(\cos^{3/2} x) + c$.
Q.18	Find the sum of the degree and the order for the following differential equation $\frac{d}{dx}[(\frac{d^2y}{dx^2})^4] = 0$. Given $\frac{d}{dx}\left[\left(\frac{d^2y}{dx^2}\right)^4\right] = 0 \quad \Rightarrow 4\left(\frac{d^2y}{dx^2}\right)^3 \times \frac{d^3y}{dx^3} = 0$. Since order and degree of the differential equation is 3 and 1 respectively. So their sum is 4. Ans. order 3 , or degree 1 $\therefore$ Degree + order = 4
Q.19	Let R_1 be a relation defined by $R_1 = \{(a,b) \mid a \geq b, a,b \in R\}$. Then R_1 is (a) An equivalence relation on R (b) Reflexive, transitive but not symmetric (c) Symmetric, Transitive but not reflexive (d)Neither transitive not reflexive but symmetric ans b OR Let $P = \{(x,y) \mid x^2 + y^2 = 1, x,y \in R\}$. Then P is (a) Reflexive (b) Symmetric

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	(c)Transitive (d) Anti-symmetric ans b
Q.20	If $M=\begin{bmatrix}1&2\\2&3\end{bmatrix}$ and $M^2-\lambda M-I_2=0$, then find the value of λ. $M^2-\lambda M-I_2=0$ $\Rightarrow\begin{bmatrix}1&2\\2&3\end{bmatrix}\begin{bmatrix}1&2\\2&3\end{bmatrix}-\begin{bmatrix}\lambda&2\lambda\\2\lambda&3\lambda\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}=O\Rightarrow\begin{bmatrix}5&8\\8&13\end{bmatrix}-\begin{bmatrix}\lambda&2\lambda\\2\lambda&3\lambda\end{bmatrix}-\begin{bmatrix}1&0\\0&1\end{bmatrix}=O$ $\Rightarrow\begin{bmatrix}5-\lambda&8-2\lambda\\8-2\lambda&13-3\lambda\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}\Longrightarrow 5-\lambda=1, 8-2\lambda=0, 13-3\lambda=1$ $\Rightarrow\lambda=4$, which satisfies all the three equations.
	PART – B (Question 21 to 26 carry 2 mark each.)
Q.21	A bag contains 5 red balls and 3 black balls three balls are drawn one by one without replacement. What is the probability that at least one of the three balls be black if the first balls are red? ans A= the first ball is red $= A=\{RRB, RBR, RBB, RRR\}$ & b = at least one of the three balls drawn is black $A\cap B=\{RRB, RBR, RBB\}$ Required probability $=P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{25/56}{35/56}=\frac{5}{7}$
Q.22	Prove that: $\tan^{-1}\left(\frac{6x-8x^3}{1-12x^2}\right)-\tan^{-1}\left(\frac{4x}{1-4x^2}\right)=\tan^{-1}2x$. $\tan^{-1}\left(\frac{3(2x)-(2x)^3}{1-3(2x)^2}\right)-\tan^{-1}\left(\frac{2(2x)}{1-(2x)^2}\right)$ let $2x=\tan\theta$ $\tan^{-1}\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)-\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$ $\Rightarrow\tan^{-1}(\tan3\theta)-\tan^{-1}(\tan2\theta)$ $\Rightarrow 3\theta-2\theta=\theta=\tan^{-1}2x$ H.P. OR Prove that : $\sin^{-1}\sin\left(\frac{33\pi}{7}\right)+\cos^{-1}\left(\cos\frac{46\pi}{7}\right)+\tan^{-1}\left(-\tan\frac{13\pi}{8}\right)+\cot^{-1}\left[\cot\left(-\frac{19\pi}{8}\right)\right]=\frac{13\pi}{7}$ .Ans. $Sin^{-1}\left(\sin\frac{33\pi}{7}\right)=\frac{2\pi}{7}$ $\cos^{-1}\left(\cos\frac{46\pi}{7}\right)=\frac{4\pi}{7}$ $\tan^{-1}\left(-\tan\frac{13\pi}{8}\right)=\frac{3\pi}{8}$ $\cot^{-1}\left\{\cot\left(-\frac{19\pi}{8}\right)\right\}=\frac{5\pi}{8}$

	$\therefore \frac{2\pi}{7}+\frac{4\pi}{7}+\frac{3\pi}{8}+\frac{5\pi}{8}=\frac{13\pi}{7}$
Q.23	If $y=\tan^{-1}\left(\frac{x}{a}\right)+\log\sqrt{\frac{x-a}{x+a}}$: prove that $\frac{dy}{dx}=\frac{2ax^2}{x^4-a^4}$. ANS : We have $y=\tan^{-1}\left(\frac{x}{a}\right)+\log\sqrt{\frac{x-a}{x+a}} \Rightarrow y=\tan^{-1}\frac{x}{a}+\frac{1}{2}[\log(x-a)-\log(x+a)]$ On differentiating w.r.t x both sides, we get : $\frac{dy}{dx}=\frac{1}{1+\frac{x^2}{a^2}}\cdot\frac{1}{a}+\frac{1}{2}\left[\frac{1}{x-a}-\frac{1}{x+a}\right]$ $\Rightarrow \frac{dy}{dx}=\frac{a}{a^2+x^2}+\frac{1}{2}\left[\frac{2a}{x^2-a^2}\right]=\frac{a}{x^2+a^2}+\frac{a}{x^2+a^2}\therefore\frac{dy}{dx}=\frac{2ax^2}{x^4-a^4}$
Q.24	Find the intervals in which the function f defined by $f(x)=-2x^3-9x^2-12x+1$ is(i) strictly increasing(ii)strictly decreasing . $f'(x)=-6(x+1)(x+2)$ $f'(x)=0\Rightarrow x=-2,\ x=-1$ $\Rightarrow$ Intervals are $(-\infty,-2)$, $(-2,-1)$ and $(-1,\infty)$ Getting $f'(x)>0$ in $(-2,-1)$ and $f'(x)<0$ in $(-\infty,-2)\cup(-1,\infty)$ $\Rightarrow$ f(x) is strictly increasing in $(-2,-1)$ and strictly decreasing in $(-\infty,2)\cup(-1,\infty)$
Q.25	If $\vec{\mathbf{a}}=\mathbf{i}+\mathbf{j}+\mathbf{k},\vec{\mathbf{b}}=\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and $\vec{\mathbf{c}}=7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$, then find the area of the parallelogram having diagonals $\vec{\mathbf{a}}+\vec{\mathbf{b}}$ and $\vec{\mathbf{b}}+\vec{\mathbf{c}}$. Ans $\vec{P}\times\vec{Q}=-8i+16j-8k\therefore A=\frac{1}{2}\left\|\vec{P}\times\vec{Q}\right\|=\sqrt{384}$ **OR** Let $\vec{a}=2\vec{i}+\vec{k},\vec{b}=\vec{i}+\vec{j}+\vec{k}$ and $\vec{c}=4\vec{i}-3\vec{j}+3\vec{k}$ be three vectors , find a vector $\vec{r}$ which satisfies $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\bullet\vec{a}=0$. Ans. $\vec{r}=\frac{1}{3}i-\frac{20}{3}j-\frac{2}{3}k$
Q.26	Find the symmetrical form, the equation of the line $2x-2y+3z-2=0, x-y+z+1=0$. ANS. $\frac{x+5}{1}=\frac{y}{1}=\frac{z-4}{0}$
	PART – C (Question 27 to 32 carry 4 mark each.)

Q.27 Find the value of 'a' for which the function f defined as f(x) = $\begin{cases} a\sin\frac{\pi}{2}(x+1)\ , & x \le 0 \\ \frac{\tan x - \sin x}{x^3}\ , & x > 0 \end{cases}$ is continuous at x = 0. Ans : f(x) = $\begin{cases} a\sin\frac{\pi}{2}(x+1)\ , & x \le 0 \\ \frac{\tan x - \sin x}{x^3}\ , & x > 0 \end{cases}$

LHL = $\lim\limits_{x\to 0^-}\left(a\sin\frac{\pi}{2}(x+1)\right)$

Applying limit a × 1 = a

RHL $= \lim\limits_{x\to 0^+}\left(\frac{\tan x - \sin x}{x^3}\right) = \lim\limits_{x\to 0^+}\frac{\sin x}{x}\cdot\frac{1}{2}\lim\limits_{x\to 0^+}\left[\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right]^2 = \lim\limits_{x\to 0^+}\frac{\sin x}{x}\cdot\lim\limits_{x\to 0^+}\frac{2\sin^2\frac{x}{2}}{x^2} = \frac{1}{2}$

LHL = RHL (its given) a = $\frac{1}{2}$

OR

If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

SOLUTION We have, $y = x^x$

or, $y = e^{\log x^x} = e^{x\log x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = e^{x\log x}\frac{d}{dx}(x\log x)$$

$$\Rightarrow \frac{dy}{dx} = x^x(1+\log x)$$

$$\Rightarrow \frac{dy}{dx} = y(1+\log x)$$

Differentiating with respect to x, we get

$$\frac{d^2}{dx^2} = y\times\frac{d}{dx}(1+\log x) + \frac{dy}{dx}\times(1+\log x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = y\times\frac{1}{x} + \frac{dy}{dx}\times(1+\log x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{dy}{dx}\left(\frac{1}{y}\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{y}{x} + \frac{1}{y}\left(\frac{dy}{dx}\right)^2 \Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$$

Q.28 Find the particular solution of the differential equation $(xdy - ydx)y.\sin\left(\frac{y}{x}\right) = (ydx + xdy)x\cos\frac{y}{x}$, given that $y = \pi$ when x = 3 .

Ans. $\log\left|\sec\frac{y}{x}\right| - \log\frac{y}{x} = 2\log x + \log\frac{2}{3\pi}$ OR $2xy\cos y/x = 3\pi$

Q.29 Evaluate : $\int_0^2 |x^2 + 2x - 3|dx$ ans:We have $x^2 + 2x - 3 = (x+3)(x-1)$ $\therefore$

$x^2 + 2x - 3 > 0$ for $x < -3$ or $x > 1$ and $x^2 + 2x - 3 < 0$ for $-3 < x < 1$ so

$$|x^2+2x-3| = \begin{cases} x^2+2x-3 & \text{for } x<-3 \text{ or } x>1 \\ -(x^2+2x-3) & \text{for } -3<x<1 \end{cases} \quad \therefore$$

$$\int_0^2 |x^2+2x-3|dx = \int_0^1 |x^2+2x-3|dx + \int_1^2 |x^2+2x-3|dx = -\int_0^1 (x^2+2x-3)dx + \int_1^2 (x^2+2x-3)dx$$

$$= -\left[\frac{x^3}{3} + x^2 - 3x\right]_0^1 + \left[\frac{x^3}{3} + x^2 - 3x\right]_1^2 = 4$$

Q.30 Testing is very important during the Covid-19 pandemic. By examining the test done at a government hospital, the probability that Covid-19 is detected when a person is actually suffering is 0.99. The probability that the doctor diagnosis incorrectly that a person has Covid-19 on the basis of test is 0.001. In a metro city, 1 in 1000 suffers from Covid-19, a person is selected at random and is diagnosed to have Covid-19.

(i) What is the probability that a person selected is not having Covid-19?

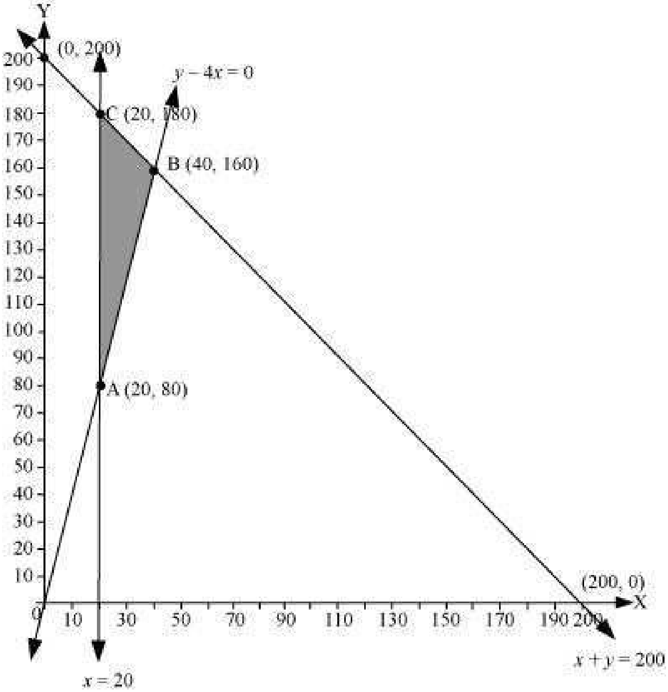
(ii) What is the probability of a person diagnosed with Covid-19.

(iii) Assume that the population of metro city is 2000000. How many persons are expected to suffer from Covid-19?

(iv) What is the probability that a person actually has Covid-19, when he is diagnosed to have Covid-19?

(v) What is the probability that the doctor diagnosis correctly that a selected person has Covid-19 on the basis of test .

	(i) (d) Let E : the person is actually suffering with Covid-19, $\bar{E}$: the person selected is not having Covid-19, A : the person is diagnosed as having Covid-19. Given that $P(E)=\frac{1}{1000}=0.001$ $\therefore$ the probability that a person selected is not having Covid-19 , $P(\bar{E})=1-0.001=0.999$. **(ii)** (d) $P(A)=P(A\mid E)P(E)+P(A\mid \bar{E})P(\bar{E})$ $\Rightarrow \quad =\frac{99}{100}\times\frac{1}{1000}+\frac{1}{1000}\times\frac{999}{1000}=0.001989$. **(iii)** (d) As 0.1% persons are expected to suffer from Covid-19. $\therefore$ No. of persons suffering from Covid-19 $=(2000000)\times 0.1\%=2000000\times\frac{1}{1000}=2000$. **(iv)** (a) By Bayes' theorem, $P(E\mid A)=\frac{P(A\mid E)P(E)}{P(A\mid E)P(E)+P(A\mid \bar{E})P(\bar{E})}$ $=\frac{\frac{99}{100}\times\frac{1}{1000}}{\frac{99}{100}\times\frac{1}{1000}+\frac{1}{1000}\times\frac{999}{1000}}$ $=\frac{990}{1989}=\frac{110}{221}$. **(v)** (c) Given that the probability that the doctor diagnosis incorrectly that a person has Covid-19 on the basis of test is 0.001. Therefore, the required probability = 1 - .001 = .999 .
Q.31	An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit? **Answer :** Let the airline sell x tickets of executive class and y tickets of economy class. The mathematical formulation of the given problem is as follows. Maximize $z = 1000x + 600y$ …(1) subject to the constraints, $x \geq 20$, $x+y \leq 200$, and $y - 4x \geq 0$, $y \geq 0$ The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A(20, 80), B(40, 160), and C(20, 180).

The values of z at these corner points are as follows.

Corner point	$z = 1000x + 600y$	
A(20,80)	68000	
B(40,160)	136000	$\rightarrow$Minimum
C(20,180)	128000	

Q.32 Check whether the relation R in R defined by $R = \{(a,b) : a \le b^3\}$ is reflexive, symmetric or transitive. Sol. $R = \{(a,b) : a \le b^3 \forall a,b \in R\}$

Reflexive : As $\frac{1}{2} > \frac{1}{8}$ for $\frac{1}{2}, \frac{1}{8} \in$ R

i.e. $\frac{1}{2} \not\le \left(\frac{1}{2}\right)^3$

$\Rightarrow$ $(a,a) \notin R \quad \forall \quad a \in R$

$\therefore$ R is not reflexive.

Symmetric: As 1<27 for 1,3 $\in$ R

i.e. (1,3) $\in$ R for 1,3 $\in$ R

But 3 $\not\le$ 1

$\Rightarrow$ (3,1)) $\notin$ R

$\therefore$ R is not symmetric.

Transitive : As 100 < 125 and 5 < 8

i.e. $100 < 5^3$ and $5 < 2^3$

	$\Rightarrow$ (100,5) $\in$ R and (5,2) $\in$ R But 100 $\nless$ 8 i.e. $100 < 2^3$ $\therefore$ (100,2) $\notin$ R $\therefore$ (100,5) $\in$ R, (5,2) $\in$ R $\Rightarrow$ (100,2) $\notin R$ $\therefore$ R is not transitive. Hence R is neither reflexive nor symmetric nor transitive
	PART – D (Question 33 to 36 carry 6 mark each.)
Q.33	If x $\neq$ y $\neq$ z and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0$ then prove that xyz(xy +yz +zx) = x+ y+ z. $\begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} = 0 \Rightarrow xyz\begin{pmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{pmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix} = 0$ $\Rightarrow xyz(x-y)(y-z)(z-x)(xy+yz+zx)-(x-y)(y-z)(z-x)(x+y+z)=0$ $(x-y)(y-z)(z-x)\{xyz(xy+yz+zx)-(x+y+z)\}=0$ $\therefore xyz(xy+yz+zx)=x+y+z$ **OR** If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ prove that $(aI+bA)^n = a^nI + na^{n-1}bA$, where I is the unit matrix of order 2 and n is a positive integer . Answer It is given that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ To show: $P(n):(aI+bA)^n = a^nI+na^{n-1}bA,\ n \in \mathbf{N}$ We shall prove the result by using the principle of mathematical induction. For n = 1, we have: $P(1):(aI+bA) = aI+ba^0A = aI+bA$ Therefore, the result is true for n = 1. Let the result be true for n = k. That is, $P(k):(aI+bA)^k = a^kI+ka^{k-1}bA$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$(aI + bA)^{k+1} = (aI + bA)^k (aI + bA)$$
$$= (a^k I + ka^{k-1}bA)(aI + bA)$$
$$= a^{k+1}I + ka^k bAI + a^k bIA + ka^{k-1}b^2A^2$$
$$= a^{k+1}I + (k+1)a^k bA + ka^{k-1}b^2A^2 \qquad ...(1)$$

$$\text{Now, } A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

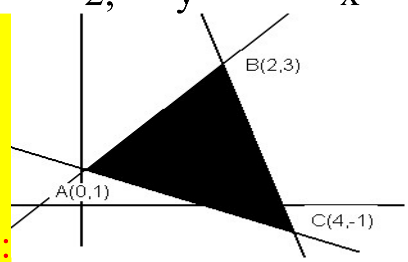
$$(aI + bA)^{k+1} = a^{k+1}I + (k+1)a^k bA + O$$
$$= a^{k+1}I + (k+1)a^k bA$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$(aI + bA)^n = a^n I + na^{n-1}bA \text{ where } A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ n \in \mathbf{N}$$

Q.34 Using integration, find the area of the triangle bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$. Ans



:

$$A_1 = \int_{-1}^{3} \frac{7-y}{2}\,dy;\ A_2 = \int_{1}^{3}(y-1)\,dy;\ A_3 = \int_{-1}^{1}(2-2y)\,dy \Rightarrow A_1 - A_2 - A_3 = 6unit^2$$

Q.35 A poster is to contain 72sq. cm. of printed matter . The margins at the top and bottom are to be 4 cm each and at the sides 2 cm each side wide. Find the dimensions if total area of the poster is minimum .

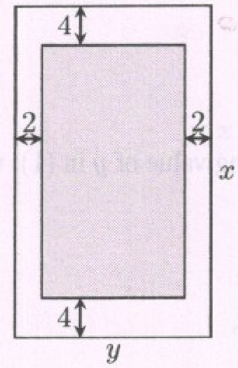
Solution: Let A denote the total area of the poster of length x cm and breadth y cm.

$\Rightarrow \qquad A = xy \qquad ...(1)$

Now, Length of printed matter $= x - 8$

and breadth of printed matter $= y - 4$

$\therefore$ Area of printed matter, $Z = (x-8)(y-4)$



Also, $Z = 72$ [Given]

$\Rightarrow \quad (x-8)(y-4) = 72$

$\Rightarrow \quad y = \dfrac{72}{x-8} + 4 \quad ...(2)$

Putting value of y in (1), we get

$$A = x\left(\frac{72}{x-8} + 4\right) = \frac{72x}{x-8} + 4x$$

Then, $\dfrac{dA}{dx} = \dfrac{72[(x-8)-x]}{(x-8)^2} + 4 = -\dfrac{576}{(x-8)^2} + 4$

Now, $\dfrac{dA}{dx} = 0 \Rightarrow -\dfrac{576}{(x-8)^2} + 4 = 0$

$\Rightarrow (x-8)^2 = 144$

$\Rightarrow x - 8 = \pm 12$

$\Rightarrow x = 20, -4$

$\Rightarrow x = 20 \quad [\because x > 0]$

Also, $\dfrac{d^2A}{dx^2} = \dfrac{1152}{(x-8)^3}$

$\Rightarrow \left(\dfrac{d^2A}{dx^2}\right)_{x=20} = \dfrac{1152}{(12)^3} > 0$

$\therefore$ By second derivative test, A is minimum when $x = 20$.

Putting $x = 20$ in (2), we get $y = 10$.

Hence, required dimensions of poster are 20 cm × 10 cm.

OR

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

Let a piece of length l be cut from the given wire to make a square.

Then, the other piece of wire to be made into a circle is of length $(28 - l)$ m.

Now, side of square $= \dfrac{l}{4}$.

Let r be the radius of the circle. Then, $2\pi r = 28 - l \Rightarrow r = \frac{1}{2\pi}(28-l)$.

The combined areas of the square and the circle (A) is given by,

$$A = (\text{side of the square})^2 + r^2$$

$$= \frac{l^2}{16} + \pi\left[\frac{1}{2\pi}(28-l)\right]^2$$

$$= \frac{l^2}{16} + \frac{1}{4\pi}(28-l)^2$$

$$\therefore \frac{dA}{dl} = \frac{2l}{16} + \frac{2}{4\pi}(28-l)(-1) = \frac{l}{8} - \frac{1}{2\pi}(28-l)$$

$$\frac{d^2A}{dl^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$$

Now, $\frac{dA}{dl} = 0 \Rightarrow \frac{l}{8} - \frac{1}{2\pi}(28-l) = 0$

$$\Rightarrow \frac{\pi l - 4(28-l)}{8\pi} = 0$$

$$\Rightarrow (\pi+4)l - 112 = 0$$

$$\Rightarrow l = \frac{112}{\pi+4}$$

Thus, when $l = \frac{112}{\pi+4}, \frac{d^2A}{dl^2} > 0$.

$\therefore$ By second derivative test, the area (A) is the minimum when $l = \frac{112}{\pi+4}$.

Q.36 Find the vector equation of the planes through the intersection of $(2\hat{i}+6\hat{j})+12=0$ and $\bar{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at a unit distance from the origin. Hence find the distance of the plane thus obtained from the plane 2x – 4y + 4z – 9 = 0 .

The required plane can be written as $\vec{r}.(2\hat{i}+6\hat{j})+12+\lambda\left[\vec{r}.(3\hat{i}-\hat{j}+4\hat{k})\right]=0$

i.e., $(2+3\lambda)x+(6-\lambda)y+4\lambda z+12=0 \ldots$(i)

As plane (i) is at unit distance from the origin (0, 0, 0) so,

$$p = \left|\frac{0(2+3\lambda)+0(6-\lambda)+0(4\lambda)+12}{\sqrt{(2+3\lambda)^2+(6-\lambda)^2+(4\lambda)^2}}\right| = 1 \qquad \therefore \lambda = \pm 2 .$$

	Replacing these values of λ in (i), we get : $2x+y+2z+3=0,\ x-2y+2z-3=0$ ...(A) Let $\pi: 2x-4y+4z-9=0$. Observe that the d.r.'s of planes (A) and π are in proportion i.e., $\frac{1}{2}=\frac{-2}{-4}=\frac{2}{4}$ which implies that these planes are parallel planes. So the distance between plane (A) and π is, $\frac{\lvert -6+9\rvert}{\sqrt{4+16+16}}=\frac{3}{6}$ or $\frac{1}{2}$ units.
	बिना शिक्षा प्राप्त किये कोई व्यक्ति अपनी परम ऊँचाइयों को नहीं छू सकता.