

Read instructions twice . Attempt your paper with honesty.

I want to know that how many concepts are clear to you.

General Instructions:

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions

Part – A:

1. It consists of two sections- I and II.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

1. It consists of three sections- III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.

Part – A (STEPS ARE REQUIRED) (24 Marks 16 + 8 = 24)

Q1. If $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 7$, then the value of x for which $f(g(x)) = 25$

Q2. What is the range of $\sin^{-1} x + \cos^{-1} x + \tan^{-1} x$.

Q3. If $A^2 - A + I = O$, then find the inverse of A .

Q4. Which two properties of matrices satisfied in given matrices..

If $A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$, then

Q5. Write the number of discontinuous functions $y(x)$ on $[-2, 2]$ satisfying $x^2 + y^2 = 4$.

Q6. if $y = (1 + x)(1 + x^2)(1 + x^4)\dots(1 + x^{2^n})$, then the value of dy/dx at $x = 0$

Q7 Find the local minimum value of the function $f(x) = \sin 4x + \cos 4x$, $0 < x < \pi/2$

Q8 The area of a right-angled triangle of the given hypotenuse is maximum what type of triangle is, give reason.

Q9 $\operatorname{Cosec} x$, $\tan x$, x^2 , $|x - 1|$ function which is neither decreasing nor increasing in $(\pi/2, 3\pi/2)$ is

Q10.

If $A = \begin{pmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{pmatrix}$ Then write the cofactors of the elements a_{21} of its 2nd row.

Q11. What is the objective function of a L.P.P. Explain it.

Q12. What is the type of function if $f : A \rightarrow B$ defined by $f(x) = 4x + 7$, $x \in R$. Explain

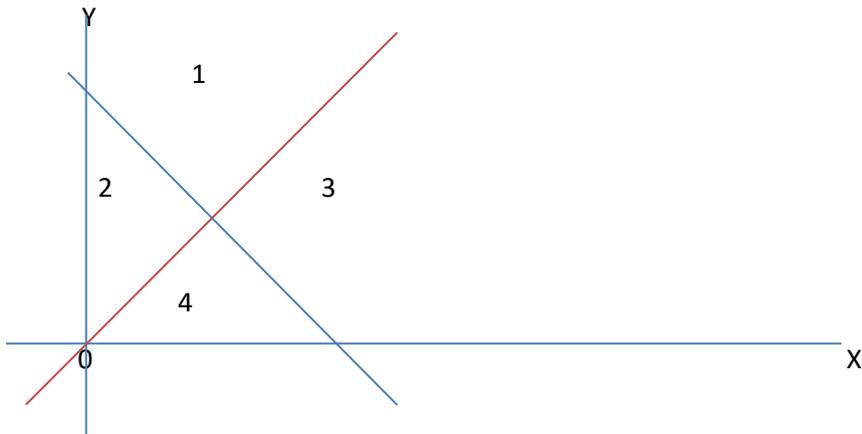
Q13. Find the derivative of $e^x / \sin x$

Q14. $\int_0^{\pi/2} \sin x^2 + \cos x^2 dx$ (sign is pie)

Q15. Show that function $f : R \rightarrow R$ is a function defined by $f(x) = 3x - 2$ is one one.

Q16. Find the value of $\tan^{-1} \left(\tan \frac{1260}{6} \right)$

Q17.



Chose the Correct option for the following (Single option is correct).

1. Objective Function of a L.P.P. is
 - (a) A constraint
 - (b) A function to be optimised
 - (c) A relation between the variables
 - (d) None of these

26. Solution set of inequation $x \geq 0$ is
 - (a) Half-plane on the left of y-axis.
 - (b) Half-plane on the right of y axis excluding the points on y-axis.
 - (c) Half-plane on the right of y-axis including the points on y-axis.
 - (d) None of these

3 8. Regions represented by equations $x \geq 0, y \geq 0$ is

- (a) first quadrant
- (b) Second quadrant
- (c) Third quadrant
- (d) Fourth quadrant

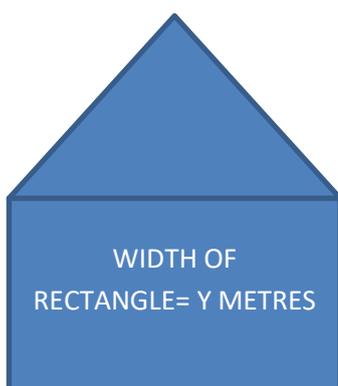
4. If the constraints in a linear programming problem are changed, then

- (a) The problem is to be re-evaluated
- (b) Solution not defined
- (c) The objective function has to be modified
- (d) The change in constraints is ignored.

5) which region excluded if $x \geq 0, y \geq 0$

- a) 1,2
- b) 2,3
- c) 3,4
- d) none of these.

Q18. A window has the shape of a rectangle surmounted by an equilateral triangle with side x metres. If the perimeter of the window is 12 cm find the dimensions of rectangle so that it may produce area of window.



a) What is length of rectangle ?

- 1) y mt
- 2) x mt
- 3) xy mt
- 4) None of these.

b) What is the perimeter of window ?

1) $x + y$

2) $6x + 2y = 1m$

3) $4x + 2y = 12m$

4) none of these.

c) What is the area of window?

1) xy

2) $xy + \frac{1}{2}$

3) $xy + \frac{\sqrt{3}}{4} x^2$

4) $xy + \sqrt{3}$

d) Window area is maximum at the condition?

1) $x = 12/55$

2) $12/55(8 + \sqrt{3})$

3) y is maximum

4) Remains same every time.

E) What is length of rectangle?

1) $138/55$

2) $123/44$

c) $\frac{138 - 24\sqrt{3}}{55}$

d) Not possible to find.

Section – B (56 MARKS) (20 + 21 + 15)

PART =1 (20 MARKS)

Q19 Solve $\int x \log 2x \, dx$

Q20. Find the maximum and minimum value of $f(x) = \sin x + \frac{1}{2} \cos 2x$ in $\left[0, \frac{\pi}{2}\right]$

Q21. If $f(x) = \begin{cases} x^3 + 3x + a & x \leq 1 \\ bx + 2 & x > 1 \end{cases}$ is every where differentiable find the value of a and b.

Q22 Show that the determinant value of a skew-symmetric matrix of odd order is always zero.

Q23. Find the value of ϕ satisfying determinant $\begin{vmatrix} 1 & 1 & \sin 3\phi \\ -4 & 3 & \cos 2\phi \\ 7 & -7 & -2 \end{vmatrix} = 0$

Q24. IF $Y = e^x (\sin x + \cos x)$ then show that $-\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

OR

Find the maximum and minimum values of $(x-3)^5 (x+1)^4$

Q25. Relation R in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ Then find the number of set of all elements to related to 3 .

Q26 Evaluate $\int_0^2 x\sqrt{2-x} \, dx$

Or $\int \log(1 + \tan x) \, dx$

Q27. Where function $f(x) = 2x^3 - 15x^2 + 36x + 4$ is maximum.

Or

Find the interval in which function

$F(x) = 2x^3 - 9x^2 + 12x + 15$ is increasing and decreasing.

Q28. Express the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 4 & 1 & 2 \end{pmatrix}$ as the sum of symmetric and skew symmetric matrix.

PART = B (21Marks)

Q29 Find the local maxima and local minima of $F(x) = 3x^4 + 4x^3 - 12x^2 + 12$

$$Q30. \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1 + \sqrt{\cot x}} dx \text{ (sign is pie)}$$

Q31. Using integration, find the area of the region bounded by the line $X - Y + 2 = 0$, $X = \sqrt{Y}$ and y axis.

Q32. A wire of length 28 cm is to be cut into two pieces. One of the piece is to be made into a square and other into a circle. What should be the length of two pieces so that the combined area of them is minimum.

OR

Minimize $Z = 13X - 15Y$, Subject of constraints

$$X + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$$

$$Q33 \int_0^{\pi/2} \log(\tan A + \cot A) = \pi \log 2$$

Q34 A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 2 m and volume is 8m³. If building of tank costs Rs 70 per sq meters for base and Rs 45 per square meter for sides. What is the cost of least expensive tank.

Q35. Using Matrices solve the following system of linear equations

$$3X - 2y + 2z = 3$$

$$x + 2y + 3z = 6$$

$$2x - y + z = 2$$

PART = C (15MARKS)

Q36. Solve the following problem graphically:

$$\text{Minimise and Maximise } Z = 3x + 9y$$

$$\text{Subject to the constraints: } x + 3y \leq 60, x + y \geq 10, x \leq y, x \geq 0, y \geq 0$$

$$Q37. \text{ Solve of the equation } \tan^{-1}(x - 1) + \tan^{-1} X + \tan^{-1}(x + 1) = \tan^{-1}(3x)$$

OR

The sum of three numbers is -1. If we multiply the second number by 2, third number by 3 and add them we get 5. If we subtract the third number from the sum of first and second numbers we get -1. Represent it by a system of equations. Find the three numbers using inverse of a matrix

Q38. Prove

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}$$

or

Find the area enclosed between circle $x^2 + y^2 = 16$ exterior to parabola $y^2 = 6x$

Q21.

Q18.