Target Mathematics by Dr. Agyat Gupta

CLASS -XII (MATHEMATICS)

Maximum Marks: 80

Time Allowed: 3 hours

CODE- AG-TMC-TS-04-N

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. If A is square matrix such that $A^2 = I$, then A^{-1} is equal to

   a. O

   b. A + I

   c. I

   d. A

2. Let A be a skew-symmetric matrix of order n then

   a. $|A| = 0$ for all $n \in N$

b. $|A| = 0$ if n is even

c. None of these

d. $|A| = 0$ if n is odd

3. If x sin (a + y) = sin y, then $\frac{dy}{dx}$ is equal to

a. $\frac{\sin a}{\sin(a+y)}$

b. $\frac{\sin^2(a+y)}{\sin a}$

c. $\frac{\sin a}{\sin^2(a+y)}$

d. $\frac{\sin(a+y)}{\sin a}$

4. Let A and B be independent events with P (A) = 0.3 and P(B) = 0.4. Find P (A|B)

a. 0.27

b. 0.3

c. 0.2

d. 0.33

5. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that all the five cards are spades?

a. $\frac{5}{1024}$

b. $\frac{3}{1024}$

c. $\frac{7}{1024}$

d. $\frac{1}{1024}$

6. Minimize Z = 5x + 10 y subject to $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$

a. Minimum Z = 310 at (60, 0)

b. Minimum Z = 320 at (60, 0)

c. Minimum Z = 330 at (60, 0)

d. Minimum Z = 300 at (60, 0)

7. If $\cot^{-1}(\sqrt{\cos\alpha})+\tan^{-1}\sqrt{\cos\alpha}=\mu$ then $\sin\mu$ is equial to

a. $\tan^2\alpha$

b. $\tan 2\alpha$

c. $\cot^2\left(\frac{\alpha}{2}\right)$

d. 1

8. $\int(\sin(\log x)+\cos(\log x))$ dx is equal to

a. log (sin x - cos x) + c

b. x sin (log x) +C

c. sin (log x) – cos (log x) + C

d. x cos (log x) + C

9. Find the vector and cartesian equations of the planes that passes through the point (1 ,4 ,6) and the normal to the plane is $\hat{i}-2\hat{j}+\hat{k}$

a. $\left[\vec{r}-\left(\hat{i}+5\hat{j}+6\hat{k}\right)\right].\left(\hat{i}-2\hat{j}+\hat{k}\right)=0; x-2y+2z+1=0$

b. $\left[\vec{r}-\left(\hat{i}+4\hat{j}+7\hat{k}\right)\right].\left(\hat{i}-2\hat{j}+\hat{k}\right)=0; x-2y+z+5=0$

c. $\left[\vec{r}-\left(\hat{i}+4\hat{j}+6\hat{k}\right)\right].\left(\hat{i}-2\hat{j}+\hat{k}\right)=0; x-2y+z+1=0$

d. $\left[\vec{r}-\left(2\hat{i}+4\hat{j}+6\hat{k}\right)\right].\left(\hat{i}-2\hat{j}+\hat{k}\right)=0; x-3y+z+1=0$

10. If $\vec{a}$ and $\vec{b}$are two collinear vectors, then which of the following are incorrect

a. $\vec{b}$=$\lambda\vec{a}$ for some scalar λ

b. both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

c. $\vec{a} = \pm\vec{b}$

d. the respective components of $\vec{a}\ and\ \vec{b}$ are not proportional

11. Fill in the blanks:

If f(x)=f (a-x), then , $\int_0^a xf(x)dx$ equal to ------

12. Fill in the blanks:

If y = x^3 + tan x, then $\frac{d^2y}{dx^2}$ is ________.

13. Fill in the blanks:

If A and B are symmetric matrices, then AB - BA is a ________ matrix.

14. Fill in the blanks:

The coordinates of the foot of the perpendicular drawn from the point (2, 5, 7) on the x-axis are given by ________.

OR

Fill in the blanks:

If the angle between the lines whose direction ratios are 2,–1 , 2 and a, 3, 5 be $45°$, then a = --------

15. Fill in the blanks:

If both f and g are defined in a nhd of 0; f(0) = 0 = g(0) and f'(0) = 8 = g'(0), then $\underset{x \to 0}{Lt} \frac{f(x)}{g(x)}$ is equal to ---------

OR

Fill in the blanks:

The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is ________.

16. If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix}$=k,then $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix}$= ------------------------

17. Evaluate $\int 3^{x+2} dx$

OR

Evaluate $\int_0^3 \frac{dx}{9+x^2}$.

18. $\int \log(\frac{1}{x} - 1) dx$ is equal to = -----------

19. Find the point at which the tangent to the curve $\sqrt{x} + \sqrt{y} = 4$ is equally inclined to the axes.

20. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$

Section B

21. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

OR

Find fog and gof, if f(x) = x + 1, g(x) = 2x + 3.

22. Find the values of x for which the function,

$f(x) = kx^3 - 9x^2 + 9x + 3$ is increasing in R

23. $If \;\; \Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & 1 & a \end{vmatrix} = -4$

$then\ find\ the\ value\ of \begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}.$

24. Find the angle between vectors $\vec{a}$ and $\vec{b}$ if $\left|\vec{a}\right| = \sqrt{3}, \left|\vec{b}\right| = 2, \vec{a}\,.\,\vec{b} = \sqrt{6}$

OR

Represent graphically a displacement of 40 km, 30^o East of North.

25. Find the vector equation of the plane that passes through the point (1,0,0) and contains the line $\vec{r} = \lambda\,\hat{j}$.

26. If A and B are two events such that P (A) = $\frac{1}{4}$ P (B) = $\frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$ find P(not A and not B).

Section C

27. Let L be the set of all lines in plane and R be the relation in L define if R = {(l_1, L_2): L_1 is $\perp$ to L_2}. Show that R is symmetric but neither reflexive nor transitive.

28. Discuss the continuity of the function f(x) at x = 1/2, when f(x) is defined as follows:

$$f(x) = \begin{cases} 1/2 + x, & 0 \leq x < 1/2 \\ 1, & x = 1/2 \\ 3/2 + x, & 1/2 < x \leq 1 \end{cases}$$

OR

Find the value of a, if the function f(x) defined by f(x) = $\begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$ is continuous at x = 2. Also, discuss the continuity of f(x) at x = 3.

29. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

30. Find $\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x} dx$

31. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

OR

Three cards are drawn at random (without replacement) from a well-shuffled pack of 52 playing cards. Find the probability distribution of number of red cards. Hence, find the mean of the distribution.

32. If a young man rides his motor-cycle at 25 km per hour, he has to spend of Rs 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per h, the petrol cost increases to Rs 5 per km and rate of pollution also increases. He has Rs 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an LPP. Solve it graphically to

find the distance to be covered with different speeds. What value is indicated in this question?

Section D

33. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then show that $A^2 - 5A + 7I = 0$ and hence find A^4 .

OR

Prove: $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ = (a -b) (b - c) (c - a) (a + b + c) $(a^2 + b^2 + c^2)$

34. Using integration, find the area in the first quadrant bounded by the curve $y = x|x|$, the circle $x^2 + y^2 = 2$ and the y-axis

35. Prove that the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.

OR

An open box with a square base is to be made out of a given quantity of cardboard of area C^2 sq units. Show that the maximum volume of box is $\frac{C^3}{6\sqrt{3}}$ cu units.

36. Find the equation of plane passing through the line of intersection of planes 2x + y - z = 3 and 5x - 3y + 4z + 9 = 0 and Parallel to line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

****

Target Mathematics

by Dr. Agyat Gupta