PRE-BOARD EXAMINATION 2020-21

AG-TMC-TS-XII-2802-12-N

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. If the function $f(x)=\begin{cases} kx^2, & \text{if} \quad x\le 2 \\ 3, & \text{if} \quad x>2 \end{cases}$ is continuous at $x=2$, then find the value of k.

OR

If $y=\log_7(\log x)$, then find $\dfrac{dy}{dx}$.

2. If $\tan^{-1}(\cot\theta)=2\theta$, then find the value of θ.

3. Find the value of $(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})\cdot(\hat{k}+\hat{i})$.

OR

If lines $\dfrac{x-1}{-3}=\dfrac{y-2}{2k}=\dfrac{z-3}{2}$ and $\dfrac{x-1}{3k}=\dfrac{y-5}{1}=\dfrac{z-6}{-5}$ are mutually perpendicular, then find the value of k.

4. If a line makes angles 90°, 135°, 45° with the X, Y, Z axes respectively, then find its direction cosines.

5. Evaluate : $\int \frac{dx}{5-8x-x^2}$

OR

Evaluate : $\int_{-\pi/4}^{\pi/4} |\sin x|\,dx$

6. For matrix $A=\begin{bmatrix} 3 & 4 & -2 \\ -4 & 5 & -3 \\ 2 & 7 & 9 \end{bmatrix}$, find $\frac{1}{2}(A-A')$. (where A' is the transpose of the matrix A)

7. Find the direction cosines of the side AC of a ΔABC whose vertices are given by A (3, 5, 4), B (–2, –2, –2) and C (3, –5, 4).

OR

Show that three points $A(-2, 3, 5)$, $B(1, 2, 3)$ and $C(7, 0, -1)$ are collinear.

8. If $A = \{1, 5, 6\}$, $B = \{7, 9\}$ and $R = \{(a, b) \in A \times B : |a - b| \text{ is even}\}$. Then write the relation R.

9. Find the degree and order of the differential equation : $5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x$.

OR

Solve the differential equation $(1 + x^2)\frac{dy}{dx} = e^y$.

10. If A and B are the points (– 3, 4, – 8) and (5, – 6, 4) respectively, then find the ratio in which yz-plane divides the line joining the points A and B.

11. If A is a square matrix such that $A^2 = A$, then find $(I + A)^3 - 7A$.

12. A line makes an angle of $\pi/4$ with each of X-axis and Y-axis. What angle does it make with Z-axis?

13. If $P = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then check whether P^{-1} exists or not.

14. Write the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$.

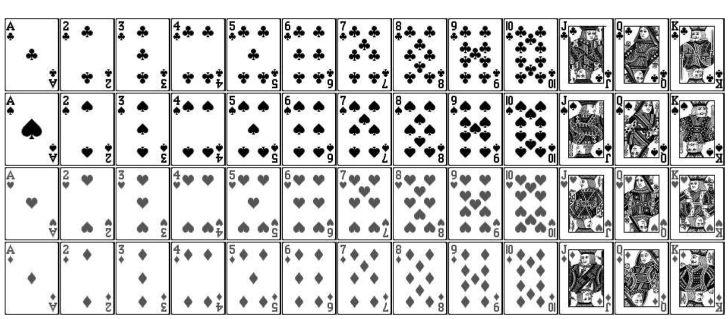
15. Let $n(A) = 4$ and $n(B) = 6$, then find the number of one-one functions from A to B.

16. A line makes 45° with OX, and equal angles with OY and OZ. Find the sum of these three angles.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A card is lost from a pack of 52 cards. From the remaining cards of pack two cards are drawn and are found to be both spades.



Based on the above information, answer the following questions :

(i) The probability of drawing two spades, given that a card of spade is missing, is

(a) $\frac{21}{425}$ (b) $\frac{22}{425}$ (c) $\frac{23}{425}$ (d) $\frac{1}{425}$

(ii) The probability of drawing two spades, given that a card of club is missing, is

(a) $\frac{26}{425}$ (b) $\frac{22}{425}$ (c) $\frac{19}{425}$ (d) $\frac{23}{425}$

(iii) Let A be the event of drawing two spades from remaining 51 cards and E_1, E_2, E_3 and E_4 be the events that lost card is of spade, club, diamond and heart respectively, then the value of $\sum_{i=1}^{4} P(A/E_i)$ is

(a) 0.17 (b) 0.24 (c) 0.25 (d) 0.18

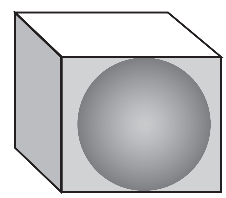
(iv) All of a sudden, missing card is found and, then two cards are drawn simultaneously without replacement. Probability that both drawn cards are aces is

(a) $\frac{1}{52}$ (b) $\frac{1}{221}$ (c) $\frac{1}{121}$ (d) $\frac{2}{221}$

(v) If two card are drawn from a well shuffled pack of 52 cards, with replacement, then probability of getting not a king in 1^{st} and 2^{nd} draw is

(a) $\frac{144}{169}$ (b) $\frac{12}{169}$ (c) $\frac{64}{169}$ (d) none of these

18. Arun got a rectangular parallelopiped shaped box and spherical ball inside it as his birthday present. Sides of the box are x, $2x$, and $x/3$, while radius of the ball is r cm.



Based on the above information, answer the following questions :

(i) If S represents the sum of volume of parallelopiped and sphere, then S can be written as

(a) $\frac{4x^3}{3}+\frac{2}{2}\pi r^2$ (b) $\frac{2x^2}{3}+\frac{4}{3}\pi r^2$

(c) $\frac{2x^3}{3}+\frac{4}{3}\pi r^3$ (d) $\frac{2}{3}x+\frac{4}{3}\pi r$

(ii) If sum of the surface areas of box and ball are given to be constant, then x is equal to

(a) $\sqrt{\frac{k^2-4\pi r^2}{6}}$ (b) $\sqrt{\frac{k^2-4\pi r}{6}}$ (c) $\sqrt{\frac{k^2-4\pi}{6}}$ (d) none of these

(iii) The radius of the ball, when S is minimum, is

(a) $\sqrt{\frac{k^2}{54+\pi}}$ (b) $\sqrt{\frac{k^2}{54+4\pi}}$ (c) $\sqrt{\frac{k^2}{64+3\pi}}$ (d) $\sqrt{\frac{k^2}{4\pi+3}}$

(iv) Relation between length of the box and radius of the ball can be represented as

(a) $x = 2r$ (b) $x=\frac{r}{2}$ (c) $x=\frac{r}{2}$ (d) $x = 3r$

(v) Minimum volume of the ball and box together is

(a) $\frac{k^2}{2(3\pi+54)^{2/3}}$ (b) $\frac{k}{(3\pi+54)^{3/2}}$ (c) $\frac{k^3}{3(4\pi+54)^{1/2}}$ (d) none of these

PART - B

Section - III

19. Find the intervals on which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ is increasing.

20. A vector $\vec{r}$ is inclined at equal angles to OX, OY and OZ. If the magnitude of $\vec{r}$ is 6 units, then find $\vec{r}$.

OR

Find the value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other.

21. If A and B are two independent events, such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$, then find the value of $P(A|A \cup B)$.

22. If $x \in [0, 1]$, then find the value of $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$.

23. Evaluate : $\int \frac{\sqrt{16 + (\log x)^2}}{x}\, dx$

OR

Evaluate : $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x}\, dx$

24. Solve the differential equation : $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$

25. The probability that it will rain on any particular day is 50%. Find the probability that it rains only on first 4 days of the week.

26. Find the derivative of $[\sqrt{1-x^2}\sin^{-1} x - x]$ w.r.t. x.

27. Find the area bounded by the curve $x^2 + y^2 = 1$ in the first quadrant.

28. Compute the adjoint of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$.

OR

If the matrix $\begin{bmatrix} 1 & a & 2 \\ 1 & 2 & 5 \\ 2 & 1 & 1 \end{bmatrix}$ is not invertible, then find the value of a.

Section - IV

29. Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f : A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, then show that f is bijective.

30. Consider $f(x) = \begin{cases} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$. If $f(x)$ is continuous at $x = 0$, then find the value of k.

31. Find the values of x for which $f(x) = (x(x-2))^2$ is an increasing function. Also, find the points on the curve, where the tangent is parallel to x-axis.

OR

An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

32. Evaluate : $\int_0^1 \{\tan^{-1} x + \tan^{-1}(1-x)\}\, dx$

33. If $y = x\log\left(\dfrac{x}{a+bx}\right)$, then prove that $x^3\dfrac{d^2y}{dx^2} = \left(x\dfrac{dy}{dx} - y\right)^2$.

34. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{e^x(\sin^2 x + \sin 2x)}{y(2\log y + 1)}$.

OR

Find the solution of the equation $\dfrac{dy}{dx} = \dfrac{y^2 - y - 2}{x^2 + 2x - 3}$.

35. Find the area bounded by $y = x^2$, the $x -$ axis and the lines $x = -1$ and $x = 1$.

Section - V

36. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

OR

Find the points on the line $\dfrac{x+2}{1} = \dfrac{y+1}{2} = \dfrac{z-3}{2}$ at a distance of 2 units from the point $(-2, -1, 3)$.

37. Solve the following linear programming problem (LPP) graphically.

Maximize $Z = 4x + 6y$

Subject to constraints:

$x + 2y \le 80,\ 3x + y \le 75\ ;\ x, y \ge 0$

OR

Solve the following linear programming problem (LPP) graphically.

Minimize $Z = 30x + 20y$

Subject to constraints : $x + y \le 8,\ x + 4y \ge 12,\ 5x + 8y \ge 20\ ;\ x, y \ge 0$

38. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, then calculate AC, BC and $(A + B)C$. Also verify that $(A + B)C = AC + BC$.

OR

Find the matrix A satisfying the matrix equation $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

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