CODE:1410- AG-TS-20-21 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

General Instructions :-

1. This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks
2. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections- I and II.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B :

1. It consists of three sections- III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section –III, 2 questions of SectionIV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

EXAMINATION 2020 -21

Time : 3 Hours | Maximum Marks : 80

CLASS – XII | **MATHEMATICS**

PART – A

Section I

All questions are compulsory. In case of internal choices attempt any one. Each question carries 1 mark .

Q.1 Give an example of matrices A and B such that $A \neq O$, $B \neq O$, $AB = O$ and $BA \neq O$.

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	We choose A, and B, such that A = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$ and B = $\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}$, then $A \neq O$, $B \neq O$. But, AB = $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$ = O and, BA = $\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\3 & 0\end{bmatrix} \neq O$
Q.2	Find a vector of magnitude 9, which is perpendicular to both the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $-2\hat{i} + \hat{j} - 2\hat{k}$. Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. Then vector perpendicular to both vectors a & b is ; $\vec{a} \times \vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\4 & -1 & 3\\-2 & 1 & -2\end{vmatrix} = (2-3)\hat{i} - (-8+6)\hat{j} + (4-2)\hat{k} = -\hat{i} + 2\hat{j} + 2\hat{k}$ $\Rightarrow \lvert\vec{a} \times \vec{b}\rvert = \sqrt{(-1)^2 + 2^2 + 2^2} = 3$ $\therefore$ Required vector $= 9\frac{\vec{a}\times\vec{b}}{\lvert\vec{a}\times\vec{b}\rvert} = \frac{9}{3}(-\hat{i} + 2\hat{j} + 2\hat{k}) = -3\hat{i} + 6\hat{j} + 6\hat{k}$
Q.3	Write the distance of the point P (x, y, z) from XOY plane. The distance of the point p(x, y, z) from the XOY plane = $\lvert z\rvert$ unit. Because the point p may be on the positive side of the z-axis or negative side of the z-axis.
Q.4	Show that the function f : R $\rightarrow$ R given by f(x) = x^3 is injective. Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$ $\Rightarrow x_1^3 = x_2^3$ $\Rightarrow x_1 = x_2$ Therefore, f is one-one function, hence f(x) = x^3 is injective. OR Let R ={ (a,b) : a, b $\in N$ and b = a +5, a < 4}. Find the domain and range of R. Given R={(a, b):b=a+5,a<4,a, b belongs N} Then dom (R) = {1, 2, 3} and range (R) = {6, 7, 8}

Q.5	If $\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\1\end{bmatrix}$, find x, y and z. Here, $\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\1\end{bmatrix}$ This product in the LHS is possible only because of the order of the first matrix is 3x3 and that of the second one is 3x1 which will result in the order of the product matrix as 3x1 which will be equal to the RHS. $\Rightarrow \begin{bmatrix}x\\-y\\-z\end{bmatrix}=\begin{bmatrix}1\\0\\1\end{bmatrix}$ ∴ x = 1, y = 0 and z = -1.
Q.6	The no. of all possible matrics of order 3 × 3 with each entry as 0 or 1 is -------- $2^9 = 512$
Q.7	Evaluate $\int \frac{3ax}{b^2+c^2x^2}dx$ Let $v = b^2 + c^2x^2$, then $dv = 2c^2\,xdx$ Therefore, $\int \frac{3ax}{b^2+c^2x^2}dx = \frac{3a}{2c^2}\int \frac{dv}{v}$ $= \frac{3a}{2c^2}\log\lvert b^2+c^2x^2\rvert + C$ OR Evaluate: $\int \sin^3 x \cos x\, dx$ Let sin x = t $\Rightarrow$ cosx dx = dt $\Rightarrow \int \sin^3 x \cos x\, dx = \int t^3\, dt$ $\Rightarrow \frac{t^4}{4}+c$ Resubstituting the value of t = sin x we get $\Rightarrow \frac{\sin^4 x}{4}+c$

Q.8

Let A be a 3 × 3 square matrix such that A (adj A) = 2 I, where I is the identity matrix. Write the value of |adj A|

Since we know

A(adj(A)) = |A|.I

2 I = |A|.I (Given A(adj A) = 2 I)

|A| = 2

Also, $|adj\ A| = |A|^{n-1}$ = 4

$= (2)^{3-1}$ |adj A| = 4

Q.9

Find the Cartesian equation of the plane $\vec{r}.[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}] = 15$

Given equation of plane is $\vec{r}.[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}] = 15$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left[(5-2t)\hat{i}+(3-t)\hat{j}+(25+t)\hat{k}\right] = 15$$

(5 - 2t)x + (3 - t)y + (25 + t)z = 15

Q.10

State the reason for the relation R on the set {1, 2, 3} given by R = {(1, 2), (2, 1)} not to be transitive.

Given set {1,2,3}

Relation R = {(1, 2), (2, 1)}.

We know that for relation R to be transitive,

$(x, y)\in R$ and $(y, z)\in R \implies (x, z)\in R$

Here, $(1, 2)\in R$ and $(2, 1)\in R$ but $(1, 1)\notin R$

Hence, Relation R is not transitive.

OR

Let the relation R be defined on N by aRb if 2a + 3b = 30. Then write R as a set of ordered pairs.

Given R = {(a, b) : 2a + 3b = 30,$\forall$ (a,b) $\in$ N}

Now according to the question (a,b)$\equiv$ 2a + 3b = 30:

$\Rightarrow$ R = {(3, 8),(6, 6),(9, 4),(12, 2)}

NOTE: 0 is a whole number that's why its not considered in this set,beacause 0 is not a natural number

Q.11

Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$

	$\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$ $\quad = \tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right)$ $= \tan^{-1}\tan\left(\pi+\frac{\pi}{8}\right)$ $\quad = \frac{\pi}{8}$
Q.12	Write the set of values of 'a' for which the function f (x) = ax + b is decreasing for all $x \in R$ f(x) = ax + b $\quad$ For f(x) to be decreasing $\quad \Rightarrow a < 0$ f'(x) = a $\quad$ f'(x) < 0 $\quad \Rightarrow a \in (-\infty, 0)$
Q.13	Find the area of the parallelogram whose adjacent sides are determined by the vectors $2\hat{i}+\hat{j}+3\hat{k}$ and $\hat{i}-\hat{j}$. $\vec{a} = 2\hat{i}+\hat{j}+3\hat{k}$ $\quad \therefore \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{vmatrix}$ $\vec{b} = \hat{i}-\hat{j}+0\hat{k}$ $= (0+3)\hat{i}-(0-3)\hat{j}+(-2-1)\hat{k}$ $= 3\hat{i}+3\hat{j}-3\hat{k}$ Area of the parallelogram $= \lvert\vec{a}\times\vec{b}\rvert$ $\quad = \sqrt{27}$ $= \sqrt{3^2+3^2+3^2}$ $\quad = 3\sqrt{3}$ sq. units
Q.14	Determine order and degree (if defined) of differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ It is given that equation is $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ We can see that the highest order derivative present in the given differential equation is $\frac{d^2y}{dx^2}$ Thus, its order is two. The given differential equation is not a polynomial equation in its derivative. Therefore, its degree is not defined. OR Find the general solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}} = 0$

	It is given that $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0 \Rightarrow \frac{dy}{dx}=-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ $\Rightarrow \frac{dy}{\sqrt{1-y^2}}=-\frac{dx}{\sqrt{1-x^2}}$ On integrating, we get, $\Rightarrow \sin^{-1}y+\sin^{-1}x=C$
Q.15	If P (A) = $\frac{3}{5}$ and P (B) = $\frac{1}{5}$ find P $(A\cap B)$ if A and B are independent events. As, A and B are independent events. Therefore, $P(A\cap B)$ = P (A). P (B)= $\frac{3}{5}\times\frac{1}{5}=\frac{3}{25}$ OR Let A and B be the events such that P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{4}$ and P(A ∩ B) = $\frac{1}{5}$ find P(A/B). P(A / B) = $\frac{P(A\cap B)}{P(B)}=\frac{(1/5)}{(1/4)}=\left(\frac{1}{5}\times\frac{4}{1}\right)=\frac{4}{5}$.
Q.16	Find the length of the perpendicular drawn from the point P(3, -4, 5) on the z-axis. Required length = $\sqrt{3^2+(-4)^2}$ = 5

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question (17-21) and (22-26). Each question carries 1 mark .

Q.17	**Matrices/Determinant**: In linear algebra, the **determinant** is a scalar value that can be computed from the elements of a square matrix and encodes certain properties of the linear transformation described by the matrix. The **determinant** of a matrix A is denoted det(A) or |A|. Using determinants/ Matrices calculate the following: Ram purchases 3 pens, 2 bags, and 1 instrument box and pays ₹ 41. From the same shop, Dheeraj purchases 2 pens, 1 bag, and 2 instrument boxes and pays ₹29, while Ankur purchases 2 pens, 2 bags, and 2 instrument boxes and pays ₹44.

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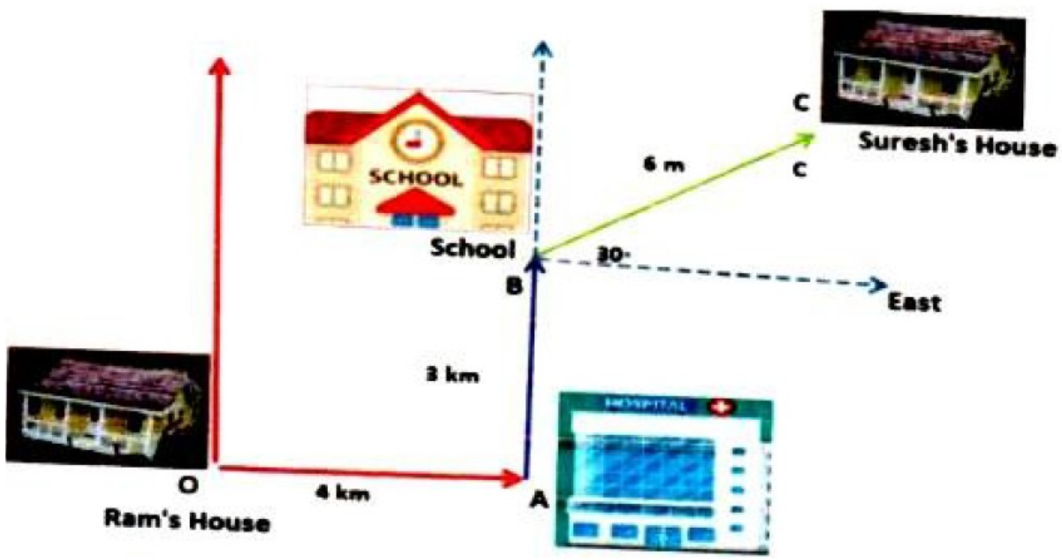
	 Read the above information and answer the following questions: i. (a) ₹ 2 ii. (c) ₹17 iii. (a) ₹7 iv. (a) ₹20 v. (a) ₹22
I	Find the cost of one pen. a. ₹ 2 b. ₹ 5 c. ₹ 10 d. ₹15
II	What are the cost of one pen and one bag? a. ₹ 12 b. ₹ 15 c. ₹ 17 d. ₹25
III	What is the cost of one pen & one instrument box? a. ₹ 7 b. ₹ 12 c. ₹ 17 d. ₹25

IV	What is the cost of one bag & one instrument box? a. ₹ 20 b. ₹ 25 c. ₹ 10 d. ₹15
V	Find the cost of one pen, one bag, and one instrument box. a. ₹ 22 b. ₹ 25 c. ₹ 20 d. ₹24
Q.18	 Ram's house is situated at Gandhi Nagar at Point O, for going to school he first travels by bus in the east. Here at Point A, a hospital is situated. From Hospital Ram takes an auto and goes 3 km in the north direction, here at point B school is situated. Suresh's house is at 30^o east, 6 km from point B. **(Refer image for information)** i. (b) 5 km ii. (a) 7 km iii. (b) $3\sqrt{3}\hat{i} + 3\hat{j}$ iv. (a) $(4+3\sqrt{2})\hat{i} + 6\hat{j}$ v. (b) 13 km

I	What is vector distance between Ram's house and school? a. 4 km b. 5 km c. 7 km d. 8 km
II	How many km Ram travels to reach school? a. 7 km b. 5 km c. 4 km d. 8 km
III	What is the vector distance from school to Suresh's home? a. $\sqrt{3}\hat{i}+\hat{j}$ b. $3\sqrt{3}\hat{i}+3\hat{j}$ c. $6\hat{i}$ d. $6\hat{j}$
IV	What is the displacement from Ram's house to Suresh house? a. $(4+3\sqrt{3})\hat{i}+6\hat{j}$ b. $4\hat{i}+6\hat{j}$ c. $13\hat{i}$ d. $13\hat{j}$
V	What is the total distance from Ram's house to Suresh's home? a. 11 km b. 13 km c. 9 km d. 5 km

PART – B

Section III

All questions are compulsory. In case of internal choices attempt any one. Each

question carries 2 mark .	
Q.19	Show that $\begin{vmatrix} \sin 10^0 & -\cos 10^0 \\ \sin 80^0 & \cos 80^0 \end{vmatrix} = 1$ $= \sin 10^o \cos 80^o + \cos 10^o \sin 80^o$ $= \sin(10^o + 80^o)$ [as sin(A + B) = sinA cosB + cosA sinB] $= sin 90^o = 1$ OR If A is a non-singular matrix, prove that $(\text{adj } A)^{-1} = \frac{1}{\lvert A\rvert} A$. We know that $A (\text{adj } A) = \lvert A\rvert I_n = (\text{adj } A) A$ $\Rightarrow \left(\frac{1}{\lvert A\rvert} A\right)(\text{adj } A) = I_n = (\text{adj } A)\left(\frac{1}{\lvert A\rvert} A\right)$ $[\because \lvert A\rvert \neq 0]$ $\Rightarrow (\text{adj } A)^{-1} = \frac{1}{\lvert A\rvert} A$
Q.20	Find the maximum and minimum value, $f(x) = 9x^2 + 12x + 2$ It is given that $f(x) = 9x^2 + 12x + 2 = (3x + 2)^2 - 2$ Now, we can see that $(3x + 2)^2 \geq 0$ for every $x \in R$ $\Rightarrow f(x) = (3x + 2)^2 - 2 \geq -2$ for every $x \in R$ The minimum value of f is attained when $3x + 2 = 0$ $3x + 2 = 0$ $\Rightarrow x = -\frac{2}{3}$ Then, Minimum value of $f = f\left(-\frac{2}{3}\right) = \left(3\left(-\frac{2}{3}\right) + 2\right)^2 - 2 = -2$ Also, since $x = -\frac{2}{3}$, is the only critical point which is a minimum, Therefore, function f does not have a maximum value.
Q.21	Show that the points whose position vectors are $-2\hat{i} + 3\hat{j},\ \hat{i} + 2\hat{j} + 3\hat{k}$ and $7\hat{i} + 9\hat{k}$ are collinear. Suppose A, B and C are the points which are represented by; $\vec{A} = -2i + 3j; \vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{C} = 7\hat{i} - \hat{k}$

	Coordinates of these points are A(-2, 3, 0); B(1, 2, 3) and C(7, 0, -1). So, $\overrightarrow{AB}$ = (1 + 2, 2 - 3, 3 - 0) = (3, -1, 3) $\overrightarrow{AC}$ = (7 + 2, 0 - 3, -1 - 0) = (9, - 3, -1) $\begin{vmatrix} 3 & -1 & 3 \\ 9 & -3 & -1 \end{vmatrix}$ = 3(1 + 9) + 1(-3 - 27) + 3(-9 + 9) = 0 Therefore, points A, B and C are collinear.
Q.22	Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is 3/5, find the probability that at least one of A and B will solve the problem. Given : Here probability of A and B that can solve the same problem is given , i.e., P(A) = $\frac{2}{3}$ and P(B) = $\frac{3}{5} \Rightarrow P(\bar{A}) = \frac{1}{3}$ and $P(\bar{B}) = \frac{2}{5}$ Also, A and B are independent . not A and not B are independent. To Find: atleast one of A and B will solve the problem Now , P(atleast one of them will solve the problem) = 1 - P(both are unable to solve) = 1 - $P(\bar{A} \cap \bar{B})$ = 1 - $P(\bar{A}) \times P(\bar{B})$ = 1 - $\left(\frac{1}{3} \times \frac{2}{5}\right) = \frac{13}{15}$ OR A and B are two independent events. The probability that A and B occur is 1/6 and the probability that neither of them occurs is 1/3. Find the probability of occurrence of two events. P(A ∩ B) = P(A) P(B) [because A and B are independent events] $\frac{1}{6} = P(A)P(B)$ $\Rightarrow P(A) = \frac{1}{6P(B)}$ ...(i) $P(\bar{A} \cap \bar{B})$ = [1 - P(A)][1 - P(B)] $\Rightarrow \frac{1}{3}$ = [1 - P(A)][1 - P(B)] $\Rightarrow \frac{1}{3} = \left[1 - \frac{1}{6P(B)}\right]$ [1 - P(B)] [on using (i)] Let P (B) = x $\Rightarrow \left(\frac{6x-1}{6x}\right)(1-x) = \frac{1}{3}$ $\Rightarrow (6x-1)(1-x) = 2x$ $\Rightarrow 6x - 6x^2 - 1 + x = 2x$ $\Rightarrow 6x^2 - 5x + 1 = 0$ $\Rightarrow$ (2x - 1)(3x - 1) = 0

	$\Rightarrow x = \frac{1}{2}$ or $x = \frac{1}{3}$ If $P(B) = \frac{1}{2}$, then $P(A) = \frac{1}{3}$ If $P(B) = \frac{1}{3}$, then $P(A) = \frac{1}{2}$
Q.23	Find the point on the curve y = x^3- 11x + 5 at which the tangent is y = x - 11. Given: Equation of the curve $y = x^3 - 11x + 5$ ....(i) Equation of the tangent y = x - 11....(ii) $\Rightarrow x - y - 11 = 0$ From equation (i), $\frac{dy}{dx} = 3x^2 - 11$ = Slope of the tangent at (x, y) But from eq. (ii), the slope of the tangent $= \frac{-a}{b} = -\frac{-1}{-1} = 1$ $\therefore\ 3x^2 - 11 = 1$ $\Rightarrow x^2 = 4$ $\Rightarrow x = \pm 2$ From equation (i), when x = 2, y = 8 - 22 + 5 = -9 And when x = -2, y = -8 + 22 + 5 = 19 Since (-2, 19) does not satisfy equation (ii), therefore the required point is (2, -9)
Q.24	If $x^x + y^x = 1$, prove that $\frac{dy}{dx} = -\left\{\frac{x^x(1+\log x)+y^x \cdot \log y}{x \cdot y^{(x-1)}}\right\}$ ATQ , $x^x + y^x = 1$ $\Rightarrow e^{\log x^x} + e^{\log y^x} = 1$ {As $e^{\text{Log a}}$ = a} $\Rightarrow e^{x \log x} + e^{x \log y} = 1$ Differentiating with respect to x using chain rule, $\frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}(1)$ $\Rightarrow e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log y}\frac{d}{dx}(x\log y) = 0$ $\Rightarrow e^{x\log x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log y^x}\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$ $\Rightarrow x^x\left[x\left(\frac{1}{x}\right) + \log x(1)\right] + y^x\left[x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y(1)\right] = 0$ $\Rightarrow x^x[1 + \log x] + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$ $\Rightarrow y^x \times \frac{x}{y}\frac{dy}{dx}$ = -[x^x(1 + log x) + y^x log y]

	$\Rightarrow \left(xy^{x-1}\right)\frac{dy}{dx}$ = -[x^x(1 + log x) + y^xlog y] $\Rightarrow \frac{dy}{dx} = -\left[\frac{x^x(1+\log x)+y^x\log y}{xy^{x-1}}\right]$ LHS=RHS Hence Proved.
Q.25	Solve (1 + x^2) $\sec^2$ ydy + 2x tan ydx = 0, given that y= $\frac{\pi}{4}$ when x = 1. Here, (1 + x^2) $\sec^2$ y dy + 2x tan y dx = 0, Given that, $y = \frac{\pi}{4}$ when x = 1 $\Rightarrow$ (1 + x^2)$\sec^2$ y dy + 2xtan y dx = 0 $\Rightarrow \frac{\sec^2 y}{\tan y}dy + \frac{2x}{1+x^2}dx = 0$ $\Rightarrow \int \frac{\sec^2 y}{\tan y}dy + \int \frac{2x}{1+x^2}dx = 0$ $\Rightarrow$ log tan y + log (1 + x^2) = log c for y = $\frac{\pi}{4}$, x, = 1 We have, 0 + log 2 = log c, c = 2, Therefore,the required particular solution is:- $\therefore \tan y\left(1+x^2\right) = 2$ OR In the differential equation show that it is homogeneous and solve it: x^2dy + y(x + y) dx = 0. The given differential equation is, x^2dy + y(x + y) dx = 0 $\Rightarrow \frac{dy}{dx} = -\frac{y(x+y)}{x^2} = -\left(\frac{y}{x}+\frac{y^2}{x^2}\right) \Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$ $\Rightarrow$ the given differential equation is a homogenous equation. The solution of the given differential equation is: Put y = vx $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow \mathrm{v} + \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = -\left(\frac{\mathrm{vx}}{\mathrm{x}} + \frac{(\mathrm{vx})^2}{\mathrm{x}^2}\right)$

	$\Rightarrow x\frac{dv}{dx}$ = -v-v^2-v = -2v -v^2 Integrating both the sides we get: $\Rightarrow \frac{dv}{2v+v^2} = -\frac{dx}{x}$ $\int \frac{dv}{2v+v^2} = -\int \frac{dx}{x} + \log\|c\|$ $\Rightarrow \int \frac{dv}{1+2v+v^2-1}$ = -ln |x| + ln|c| $\Rightarrow \int \frac{dv}{(v+1)^2-1^2}$ + ln|x| = ln|c| $\Rightarrow \frac{1}{2}\ln\left\|\frac{v+1-1}{v+1+1}\right\|$ + ln|x| = ln|c| $\Rightarrow \ln\left\|\frac{v+1-1}{v+1+1}\right\|$ + 2 ln|x| = 2 ln|c| Re-substituting the value of y = vx we get $\Rightarrow \ln\left\|\frac{\frac{y}{x}}{\frac{y}{x}+2}\right\|$ + lnx^2 = ln|c|^2 $\Rightarrow \ln\left\|\frac{y}{y+2x}\right\|$ + Inx^2 = In|c|^2 $\Rightarrow x^2y = c^2(y+2x)$, which is the required solution.
Q.26	Evaluate: $\int \frac{\sin^4 x}{\cos^8 x}dx$ Let I = $\int \frac{\sin^4 x}{\cos^8 x}dx$. Then, we have $I = \int \frac{\frac{\sin^4 x}{\cos^4 x}}{\frac{\cos^8 x}{\cos^4 x}}dx$ [Dividing numerator and denominator by cos^4 x] $\Rightarrow$ I = $\int$ tan^4 x sec^4 x dx $\Rightarrow$ I = $\int$ tan^4 x (1 + tan^{2}x) sec^2 x dx = $\int$ tan^4 x (1 + tan^{2}x) sec^2 x dx Put tan x - t and secz x dx = dt, we get I = $\int$t^4 (1 + t^2) dt = $\frac{t^5}{5} + \frac{t^7}{7} + C = \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C$
Q.27	Find values of k if area of triangle is 35 square units having vertices as (2, -6), (5, 4), (k, 4). Area of triangle = 35 units $\Rightarrow \frac{1}{2}\begin{bmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{bmatrix} = \pm 35$ Expanding along row Ist, $\Rightarrow \frac{1}{2}$[2(4 - 4)+ 6(5 - k)+1(20 - 4k)] = $\pm$35 $\Rightarrow \frac{1}{2}$[30 - 6k + 20 - 4k] = $\pm$35 $\Rightarrow \frac{1}{2}$[50 - 10k] =$\pm$35 $\Rightarrow$ 25 - 5k = $\pm$35

$\Rightarrow$ 25 - 5k = 35 or 25 - 5k = -35

$\Rightarrow$ -5k = 10 or 5k = 60

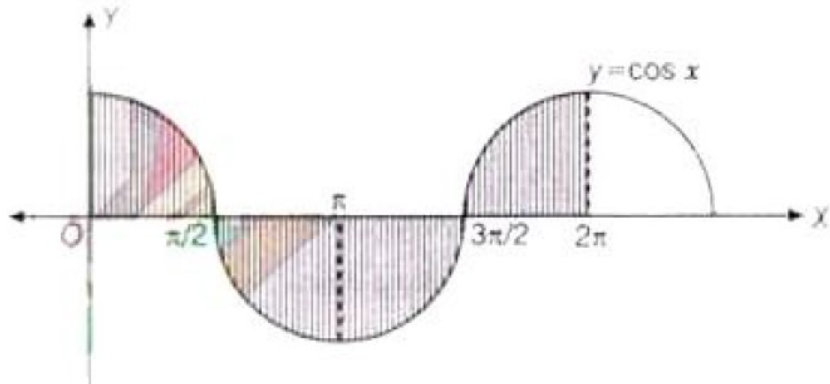
$\Rightarrow$ k = -2 or k = 12

Q.28

Find the area bounded by the curve y = 2 cos x and the x - ax is from x = 0 to $x = 2\pi$.

Required area of shaded region $= \int_0^{2\pi} 2\cos x dx$

$= \int_0^{2\pi} 2\cos x dx + \left|\int_{\pi/2}^{3\pi/2} 2\cos x dx\right| + \int_{3\pi/2}^{2\pi} 2\cos x dx$



$= 2[\sin x]_0^{\frac{\pi}{2}} + \left|2(\sin x)_{\pi/2}^{3\pi/2}\right| + 2[\sin x]_{3\pi/2}^{2\pi}$

= 2 + 4 + 2 = 8 sq units

Section IV

All questions are compulsory. In case of internal choices attempt any one. Each question carries 3 mark .

Q.29

Evaluate: $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$.

Suppose $\cos^{-1}\frac{4}{5} = \theta$, where $\theta \in [0, \pi]$

Then, $\cos\theta = \frac{4}{5}$

Since, $\theta \in [0, \pi] \Rightarrow \frac{1}{2}\theta \in \left[0, \frac{\pi}{2}\right] \Rightarrow \sin\frac{1}{2}\theta > 0$

$\therefore \sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \sin\frac{1}{2}\theta = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\left(\frac{4}{5}\right)}{2}} = \frac{1}{\sqrt{10}}$

Q.30

If $\int_0^a \sqrt{x}\, dx = 2a\int_0^{\pi/2} \sin^3 x\, dx$, find the value of integral $\int_a^{a+1} x dx$

We have,

$\int_0^a \sqrt{x}\, dx = \frac{2}{3}\left[x^{3/2}\right]_0^a = \frac{2}{3}a^{3/2}$ ....(i)

Let $I = \int_0^{\pi/2} \sin^3 x dx$, then

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	$I=\int_0^{\pi/2}\frac{3\sin x-\sin 3x}{4}dx=\frac{1}{4}\int_0^{\pi/2}(3\sin x-\sin 3x)dx=\frac{1}{4}\left[-3\cos x+\frac{1}{3}\cos 3x\right]_0^{\pi/2}$ $\Rightarrow \quad I=\frac{1}{4}\left[\left(-3\cos\frac{\pi}{2}+\frac{1}{3}\cos\frac{3\pi}{2}\right)-\left(-3+\frac{1}{3}\right)\right]$ $=\frac{1}{4}\left[0-\left(-3+\frac{1}{3}\right)\right]=\frac{1}{4}\left[3-\frac{1}{3}\right]=\frac{2}{3}\ldots\ldots(ii)$ It is given that $\int_0^a\sqrt{x}\,dx=2a\int_0^{\pi/2}\sin^3 x\,dx$ $\Rightarrow \frac{2}{3}a^{3/2}=2a\left(\frac{2}{3}\right)$ $\Rightarrow a^{3/2}=2a\Rightarrow a^3=4a^2\Rightarrow a^2(a-4)=0$ $\Rightarrow a=0,4$ [Using(i) and (ii)] When a = 4, we get $\int_a^{a+1}x\,dx=\int_4^5 x\,dx=\left[\frac{x^2}{2}\right]_4^5=\frac{25}{2}-\frac{16}{2}=\frac{9}{2}$ When a = 0, we get $\int_a^{a+1}x\,dx=\int_0^1 x\,dx=\left[\frac{x^2}{2}\right]_0^1=\frac{1}{2}-0=\frac{1}{2}$ Hence, $\int_a^{a+1}x\,dx=\frac{9}{2}\ or,\ \frac{1}{2}$ OR Evaluate the definite integral: $\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$ We have, I = $\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right)dx$ I = $\int_1^2\frac{1}{x}\cdot e^{2x}$ - $\int_1^2\frac{1}{2x^2}\cdot e^{2x}dx$ $\Rightarrow$ I = I_1 - I_2 Now, I_1 = $\int_1^2\frac{1}{x}e^{2x}$ (By parts we have) $\Rightarrow$ I_1 = $[\frac{1}{x}]_1^2\cdot\int_1^2 e^{2x}dx$ - $\int_1^2 -\frac{1}{x^2}\frac{e^{2x}}{2}dx$ $\Rightarrow$ I_1 = $\left[\frac{1}{x}\cdot\frac{e^{2x}}{2}\right]_1^2$ + $\int_1^2\frac{1}{2x^2}e^{2x}dx$

$\Rightarrow I_1 = \left[\frac{1}{2x}e^{2x}\right]_1^2 + I_2$

As, $I = I_1 - I_2$

$\Rightarrow I = \left[\frac{1}{2x}e^{2x}\right]_1^2 - I_2 + I_2$

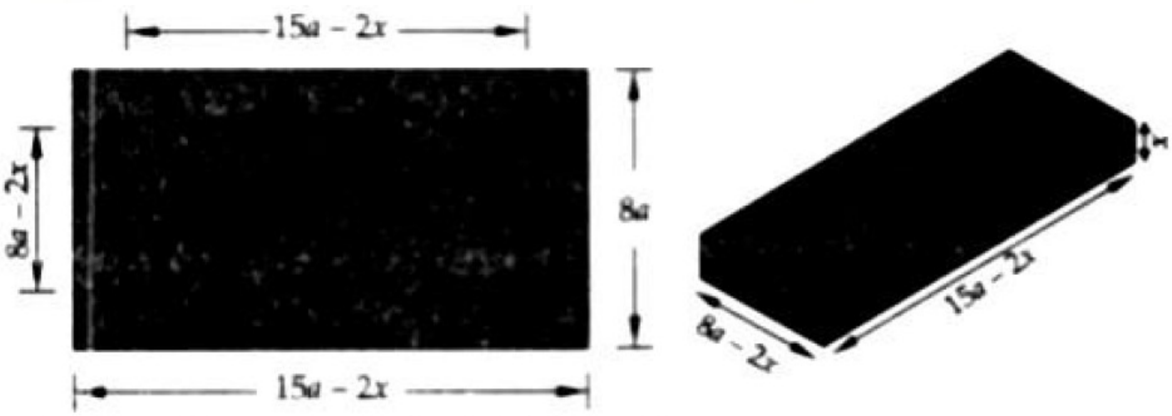
$\Rightarrow I = \left[\frac{1}{2x}e^{2x}\right]_1^2 = \frac{1}{2}\left[\frac{1}{2}e^4 - e^2\right]$

$\Rightarrow I = \frac{1}{4}e^2(e^2 - 1)$

Q.31 A rectangular sheet of fix perimeter with sides having their lengths in the ratio 8:15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed square is 100 square units, the resulting box has maximum volume. Find the length of the sides of the rectangular sheet.

Suppose the sides of rectangular sheet be 8a and 15a units respectively. Let the length of each side of the squares of same size removed from each comer of the sheet be x units. Then, the dimensions of the open box, formed by folding up the flaps,are: Length = 15a - 2x, breadth = 8a - 2x, height = x

Suppose V be the volume of the box formed. Then,



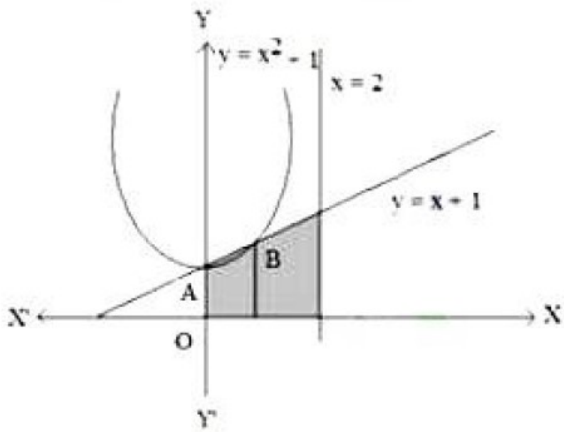
$V = (15a - 2x)(8a - 2x)x$

$\Rightarrow V = 120a^2x - 46ax^2 + 4x^3$ diff w

The critical points V are given by $\frac{dV}{dx} = 0$.

$\because \frac{dV}{dx} = 0$

$\Rightarrow 120a^2 = -92ax + 12x^2 \Rightarrow 30a^2 - 23ax + 3x^2 = 0 \Rightarrow (5a - 3x)(6a - x) = 0 \Rightarrow x = 6a, x = \frac{5a}{3}$

	But x = 6a is not possible as for x = 6a breadth = 8a - 12a = -4a, which is not possible, thus, $\frac{5a}{3}$. When x = $\frac{5a}{3}$, $\frac{d^2V}{dx^2}$ = -92a + 40a = -52a < 0, Therefore, V is maximum when x = $\frac{5a}{3}$. It is given that total area of four squares removed from each corner of the sheet is 100 sq. units. $\because 4x^2 = 100 \Rightarrow x^2 = 25 \Rightarrow \frac{25a^2}{9} = 25 \Rightarrow a^2 = 9 \Rightarrow a = 3$ Therefore, the dimensions of the sheet are 15a = 45 and 8a = 24.
Q.32	Using integration, find the area of the region given below: $\left\{(x,y) : 0 \leqslant y \leqslant x^2+1, 0 \leqslant y \leqslant x+1, 0 \leqslant x \leqslant 2\right\}$  $y = x^2 + 1$ $y = x + 1$ $x = 2$ Area $= \int_0^1 (x^2+1)\,dx + \int_1^2 (x+1)\,dx$ $= \left[\left(\frac{x^3}{3}+x\right)\right]_0^1 + \left[\left(\frac{x^2}{2}+x\right)\right]_1^2$ $= \left[\left(\frac{1}{3}+1\right)-0\right] + \left[(2+2)-\left(\frac{1}{2}+1\right)\right]$ $= \frac{23}{6}$ sq units.
Q.33	If sin y = x sin (a + y), prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ $\sin y = x \cdot \sin(a+y)$ $\Rightarrow x = \frac{\sin y}{\sin(a+y)}$ ...(i)

	Differentiating equation (i) w.r.t. y, we get $\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y\cos(a+y)}{\sin^2(a+y)}$ $\frac{dx}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$ $\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ OR Discuss the continuity of the f(x) at the indicated point: f(x) = |x| + |x - 1| at x = 0, 1 Given function is: f(x) = |x| + |x - 1| We have, (LHL at x = 0) = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$ $= \lim_{h \to 0}$ [(0 - h| + |0 - h -1|] = 1 (RHL at x = 0) = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h)$ $= \lim_{h \to 0}$ [(0 + h| + |0 + h - 1|] = 1 Also, f(0) = |0| + |0 - 1| = 0 + 1 = 1 Now, (LHL at x = 1) = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)$ $= \lim_{h \to 0}$ (|1 - h| + |1 - h - 1|) = 1 + 0 = 1 (RHL at x = 1) = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h)$ $= \lim_{h \to 0}$ (|1 + h| + |1 + h - 1|) = 1 + 0 = 1 Also, f(1) = |1| + |1 - 1| = 1 + 0 = 1 $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$ and $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$ Hence, f(x) is continuous at x = 0, 1

Q.34

Find the equations of the planes parallel to the plane x - 2y + 2z - 3 = 0 which is at a unit distance from the point (1, 2, 3).

The equation of a plane parallel to the plane x - 2y + 2z - 3 = 0 is x - 2y + 2z + λ = 0

Distance of plane (i) from point (1, 2, 3) is given by $\left|\frac{1-2\times2+2\times3+\lambda}{\sqrt{1^2+(-2)^2+2^2}}\right| = \left|\frac{\lambda+3}{3}\right|$.

But, this distance is given to be unity.

$\therefore |\lambda+3| = 3 \Rightarrow \lambda+3 = \pm 3 \Rightarrow \lambda = 0 \text{ or, } \lambda = -6$

Putting the values of X in (i), we obtain x - 2y + 2z = 0 and x - 2y + 2z - 6 = 0 as the equations of the required planes.

OR

Reduce the equation $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k})$ + 6 = 0 to the normal form and hence find the length of the perpendicular from the origin to the plane.

Given equation of plane is,

$\vec{r}\cdot(\hat{i}-2\hat{j}+2k) + 6 = 0$

$\vec{r}\cdot(\hat{i}-2\hat{j}+2k) = -6$

Multiplying both the sides by (- 1),

$\vec{r}\cdot(-\hat{i}+2\hat{j}-2\hat{k}) = 6$

$\vec{r}\cdot\vec{n} = 6$ ...(i)

Here, $\vec{n} = -\hat{i}+2\hat{j}-2\hat{k}$

$|\vec{n}| = \sqrt{(-1)^2+(2)^2+(-2)^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9}$

= 3

Dividing equation (i) by $|\vec{n}|$ = 3 both the sides,

$\vec{r}\cdot\frac{\vec{n}}{|\vec{n}|} = \frac{6}{|\vec{n}|}$

$\vec{r}\cdot\frac{1}{3}(-\hat{i}+2\hat{j}-2\hat{k}) = \frac{6}{3}$

$\vec{r}\left(-\frac{\hat{i}}{3}+\frac{2}{3}\hat{j}-\frac{2}{3}\hat{k}\right) = 2$ ...(ii)

We know that, equation of a plane at distance d from origin and normal to unit vector $\hat{n}$ is

$\vec{r} \cdot \hat{n}$ = d ...(iii)

Comparing equation (ii) and (iii),

d = 2

$$\hat{n} = -\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$$

So, normal form of equation of plane is,

$$\vec{r} \cdot \left(-\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2$$

Length of perpendicular from origin to plane

d = 2 unit.

Q.35

Minimize z = 3x + 5y subject to the constraints x + 2y $\leq$ 2000, x + y $\leq$ 1500, y $\leq$ 600, x $\geq$ 0 and y $\geq$0

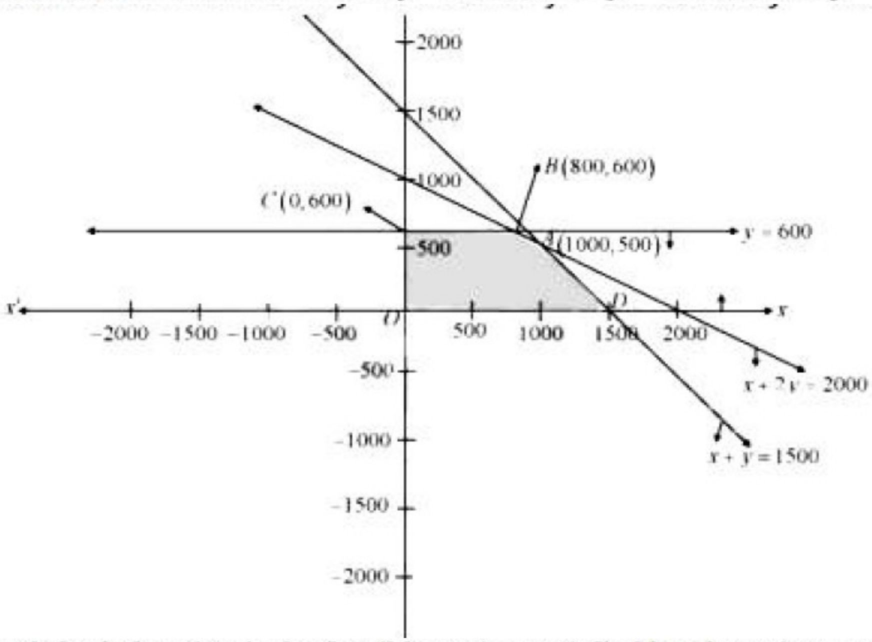
Given,

Objective function, Maximize z = 3x + 5y

subject to the constraints

x + 2y $\leq$ 2000, x + y $\leq$ 1500, y $\leq$ 600, x $\geq$ 0 and y $\geq$ 0

Now, draw the line x + 2y = 2000, x + y = 1500, and y = 600



and shaded region is the feasible region satisfied by above inequalities. Here, the feasible region is bounded.

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Corner points (x, y)	Z = 3x + 5y
(0, 0)	0
(1500, 0)	3.1500 + 5.0 = 4500
(1000, 500)	3.1000 + 5.500 = 5500
(0, 500)	0 + 500.5 = 2500

Hence the maximum value of z is 5500, which occurs at A(1000, 500).

Section V

All questions are compulsory. In case of internal choices attempt any one. Each question carries 5 mark .

Q.36

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A + 4I = 0$ and hence find A^{-1}

Given: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$$\therefore A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$\text{Now } A^3 = A^2.A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

L.H.S. $= A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22-36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} + \begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & 14 \end{bmatrix} + \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Now, to find A^{-1}, multiplying $A^3 - 6A^2 + 9A - 4I^{-1} = 0.A^{-1}$ by A^{-1}

$\Rightarrow A^3A^{-1} - 6A^2A^{-1} + 9AA^{-1} - 4I . A^{-1} = 0A^{-1}$

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$\Rightarrow 4A^{-1} = A^2 - 6A + 9I$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

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$$\Rightarrow 4A^{-1} = \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

OR

Find x, y, z if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Also, $A' = A^{-1}$

$\Rightarrow AA' = AA^{-1}$ [$\because AA^{-1} = I$]

$\Rightarrow AA' = I$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2 \\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2 \\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2-z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 2y^2 - z^2 = 0 \Rightarrow 2y^2 = z^2$

$\Rightarrow 4y^2 + z^2 = 1$

$\Rightarrow 2.z^2 + z^2 = 1$

$z = \pm\frac{1}{\sqrt{3}}$

$\therefore y^2 = \frac{z^2}{2} \Rightarrow y = \pm\frac{1}{\sqrt{6}}$

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	Also, $x^2 + y^2 + z^2 = 1$ $\Rightarrow x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3}$ $= 1 - \frac{3}{6} = \frac{1}{2}$ $\Rightarrow x = \pm\frac{1}{\sqrt{2}}$ $\therefore x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}$ and $z = \pm\frac{1}{\sqrt{3}}$
Q.37	There are three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and take out a coin. If the coin is of gold, then what is the probability that the other coin in box is also of gold? Let us define the events as E_1: Box I is selected E_2: Box II is selected E_3 · Box III is selected A: The drawn coin is a gold coin Since events E_1, E_2 and E_3 are mutually exclusive and exhaustive events. $\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ Now, $P(A/E_1)$ = Probability that a gold coin is drawn from box I $= \frac{2}{2} = 1$ [$\because$ box I contain both gold coins] $P(A/E_2)$ = Probability that a gold coin is drawn from box II=0[$\because$ box II has both silver coins] and $P(A/E_3)$ = Probability that a gold coin is drawn from box III $= \frac{1}{2}$ [$\because$ box III contains 1 gold and 1 silver coin] The probability that other coin in box is also of gold = The probability that the drawing gold coin from bag I $= P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{[P(E_1)P(A/E_1)+P(E_2)P(A/E_2)]}$ [by Baye' s theorem]

	$= \dfrac{\frac{1}{3}\times 1}{\left(\frac{1}{3}\times 1\right)+\left(\frac{1}{3}\times 0\right)+\left(\frac{1}{3}\times\frac{1}{2}\right)} = \dfrac{1}{1+0+\frac{1}{2}} = \dfrac{1}{3/2} = \dfrac{2}{3}$ Hence, the required probability is $\frac{2}{3}$. OR Three bags contain a number of red and white balls as follows: Bag 1 : 3 red balls, Bag 2 : 2 red balls and 1 white ball and Bag 3 : 3 white balls. The probability that bag a will be chosen and a ball is selected from it is $\frac{1}{6}$. What is the probability that i. a red ball will be selected? ii. a white ball is selected? Bag I: 3 red balls and 0 white ball. Bag II: 2 red balls and 1 white ball. Bag III: 0 red ball and 3 white balls. Let E_1, E_2 and E_3 be the events that bag I, bag II and bag III is selected and a ball is chosen from it. $P(E_1) = \frac{1}{6}, P(E_2) = \frac{2}{6}$ and $P(E_3) = \frac{3}{6}$ i. Let E be the event that a red ball is selected. Then, Probability that red ball will be selected. $P(E) = P(E_1)\cdot P\left(\frac{E}{E_1}\right) + P(E_2)\cdot\left(\frac{E}{E_2}\right) + P(E_3)\cdot P\left(\frac{E}{E_3}\right)$ $= \frac{1}{6}\cdot\frac{3}{3} + \frac{2}{6}\cdot\frac{2}{3} + \frac{3}{6}\cdot 0$ $= \frac{1}{6} + \frac{2}{9} + 0$ $= \frac{3+4}{18} = \frac{7}{18}$ ii. Let F be the event that a white ball is selected. $\therefore P(F) = P(E_1)\cdot P\left(\frac{F}{E_1}\right) + P(E_2)\cdot P\left(\frac{F}{E_2}\right) + P(E_3)\cdot P\left(\frac{F}{E_3}\right)$ $= \frac{1}{6}\cdot 0 + \frac{2}{6}\cdot\frac{1}{3} + \frac{3}{6}\cdot 1 = \frac{1}{9} + \frac{3}{6} = \frac{11}{18}$ **Note:** P(F) = 1 - P(E) $= 1 - \frac{7}{18} = \frac{11}{18}$ [since, we know that P(E) + P(F) = 1]
Q.38	A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of ΔABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

Suppose the equation of the variable plane is

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$....(i)

Above plane (i) meets the X-axis, Y-axis and Z-axis at the point A(a, 0, 0), B (0, b, 0) and C(0, 0, c) respectively and let (α, β, γ) be the coordinates of the centroid of ΔABC.

$\Rightarrow \alpha=\frac{a+0+0}{3}, \beta=\frac{0+b+0}{3}$ and $\gamma=\frac{0+0+c}{3}$ $\Rightarrow \alpha=\frac{a}{3}, \beta=\frac{b}{3}$ and $\gamma=\frac{c}{3}$

$\Rightarrow a=3\alpha, b=3\beta$ and $c=3\gamma$ .....(ii)

$\therefore$ 3p = length of the perpendicular from (0, 0, 0) to the plane (i)

$\Rightarrow 3p=\frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}} \Rightarrow 3p=\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$

$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{9p^2}$

$\Rightarrow \frac{1}{9\alpha^2}+\frac{1}{9\beta^2}+\frac{1}{9\gamma^2}=\frac{1}{9p^2}$ [using Equation (ii)]

$\Rightarrow \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{1}{p^2}$

The locus of the centroid is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$

OR

Show that the lines $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and $\vec{r}=(4\hat{i}+\hat{j})+\mu(5\hat{i}+2\hat{j}+\hat{k})$ intersect. Also, find their point intersection.

Here, it is given that

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$

$\vec{r}=(4\hat{i}+\hat{j})+\mu(5\hat{i}+2\hat{j}+\hat{k})$

$\overrightarrow{a_1}=i+2\hat{j}+3\hat{k}$

$\overrightarrow{b_1}=2\hat{i}+3\hat{j}+4\hat{k}$

$\overrightarrow{a_2}=4\hat{i}+\hat{j}$

$\overrightarrow{b_2}=5\hat{i}+2\hat{j}+\hat{k}$

$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 5 & 2 & 1\end{vmatrix}=\hat{i}(3-8)-\hat{j}(2-20)+\hat{k}(4-15)$

$\therefore \overrightarrow{b_1}\times\overrightarrow{b_2}=-5\hat{i}+18\hat{j}-11\hat{k}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = (4-1)\hat{i} + (1-2)\hat{j} + (0-3)\hat{k}$

$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$

$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$

$= -15 - 18 + 33$

$= 0$

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$\Rightarrow \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$\therefore \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$

General point on L1 is

$x_1 = 2\lambda + 1, y_1 = 3\lambda + 2, z_1 = 4\lambda + 3$

Suppose, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Thus, point P satisfies the equation of line $\vec{L}_2$.

$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$

$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Thus, $x_1 = 2(-1) + 1, y_1 = 3(-1) + 2, z_1 = 4(-1) + 3$

$\Rightarrow x_1 = -1, y_1 = -1, z_1 = -1$

Therefore, point of intersection of given lines is (-1, -1, -1).

" THE TWO MOST POWERFUL WARRIORS ARE PATIENCE AND TIME "

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