PRE-BOARD EXAMINATION 2020-21

CODE-AG-TMC-TS-XII-08-2102

MATHEMATICS

Time allowed : 3 hours | ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part -A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Write the cofactor of the element a_{31} in $A = \begin{pmatrix} 3 & 2 & 6 \\ 5 & 0 & 7 \\ 3 & 8 & 5 \end{pmatrix}$.

OR

If A is a square matrix of order 3 and $|2A| = k|A|$, then find the value of k.

2. Evaluate : $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

3. Find the integrating factor of the differential equation $\left\{\frac{e^{-2\sqrt{x}}}{\sqrt{x}} + \frac{y}{\sqrt{x}}\right\}\frac{dx}{dy} = 1 (x \neq 0)$.

OR

Find order and degree of the equation $\left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + 4y = \sin x \cdot$

4. If $f(x) = x^2 - 4x + 1$, find $f(A)$, where $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$.

5. Find the unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

OR

Find the projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$ on $2\hat{i} + 6\hat{j} + 3\hat{k}$.

6. Check whether the function $f(x) = x^3 - 3x^2 - x$ is one-one or not?

7. Evaluate : $\int_1^3 (x-1)(x-2)(x-3)dx$

OR

Evaluate : $\int_{-\pi}^{\pi} x^{10} \sin^7 x \; dx$

8. Check whether the lines having direction ratios $(\sqrt{3}-1, -\sqrt{3}-1, 4)$ and $(-\sqrt{3}+1, \sqrt{3}+1, -4)$ are perpendicular to each other.

9. If the vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 4x\hat{j} + y\hat{k}$ are parallel, then the values of x and y.

OR

Find the point which divides the line segment joining the points (–2, 3, 5) and (1, 2, 3) in the ratio 2 : 3 externally.

10. Find vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$.

11. Let $A = \{1, 2, 3, 4\}$. Show that $f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$ is a bijection from A to A?

12. If $|\mathbf{a}| = 4, |\mathbf{b}| = 2$ and the angle between a and b is $\frac{\pi}{6}$, then $(\vec{a} \times \vec{b})^2$ is equal to ---------

13. Find the area of the region bounded between the line $x = 2$ and the parabola $y^2 = 8x$.

14. Find equation of a line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

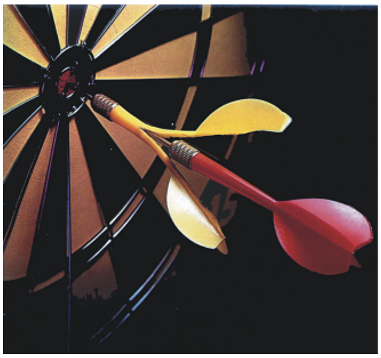
15. If $A = \begin{bmatrix} 2 & 2 \\ -3 & 1 \\ 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 1 & 3 \\ 0 & 4 \end{bmatrix}$, such that $A + B + C$ is a zero matrix, then find the matrix C.

16. If the line joining (2, 3, –1) and (3, 5, –3) is perpendicular to the line joining (1, 2, 3) and (3, 5, λ), then find the value of λ.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A teacher arranged a surprise game for students of a classroom having 5 students, namely Amit, Aruna, Eklavya, Yash and Samina. He took a bag containing tickets numbered 1 to 11 and told each student to draw two tickets without replacement.



(i) Probability that both ticket drawn by Amit shows even number, is

(a) 1/11 (b) 2/11 (c) 3/11 (d) 4/11

(ii) Probability that both tickets drawn by Aruna shows odd number, is

(a) 1/11 (b) 2/11 (c) 3/11 (d) 4/11

(iii) When tickets are drawn by Eklavya, find the probability that number on one ticket is a multiple of 4 and on other ticket is a multiple 5.

(a) 4/55 (b) 6/55 (c) 7/55 (d) None of these

(iv) When tickets are drawn by Yash, find the probability that number on one ticket is a prime number and on other ticket is a multiple of 4 .

(a) 3/11 (b) 5/11 (c) 6/11 (d) 2/11

(v) When tickets are drawn by Samina, find the probability that first ticket drawn shows an even number and second ticket drawn shows an odd number.

(a) 2/11 (b) 3/11 (c) 5/11 (d) 8/11

18. An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 4000 *l* of water, that comes out from a tube well.

Based on above information, answer the following questions.

(i) If the length, width and height of the open tank be x, x and y m respectively, then surface area of tank is given by

(a) $S = x^2 + 2xy$ (b) $S = 2x^2 + 4xy$ (c) $S = 2x^2 + 2xy$ (d) $S = 2x^2 + 8xy$

(ii) The relation between x and y is

(a) $x^2y = 4$ (b) $xy^2 = 4$ (c) $x^2y^2 = 4$ (d) $xy = 4$

(iii) The outer surface area of tank will be minimum when depth of tank is equal to

(a) half of its width (b) its width (c) $\left(\frac{1}{4}\right)^{th}$ of its width (d) $\left(\frac{1}{3}\right)^{rd}$ of its width

(iv) The cost of material will be least when width of tank is equal to

(a) half of its depth (b) twice of its depth (c) $\left(\frac{1}{4}\right)^{th}$ of its depth (d) thrice of its depth

(v) If cost of aluminium sheet is ₹ 360/m^2, then the minimum cost for the construction of tank will be

(a) ₹ 2320 (b) ₹ 3320 (c) ₹ 4320 (d) ₹ 5320

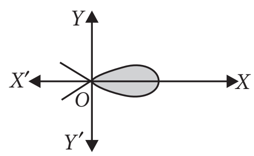
PART - B

Section - III

19. Find the equations of the tangent and the normal to the curve $y = x^3$ at the point $P(1, 1)$.

20. Express $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, $x \in \left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$ in the simplest form.

21. Find the area of region bounded by the curve $y^2 = x(1 - x)^2$, shown in following figure.



22. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. If an orator is chosen at random, find the probability that a male person is selected. Assume that there are equal number of men and women.

23. Evaluate : $\int \frac{1}{\sqrt{1-\sin x}}\,dx$

OR

Evaluate : $\int \frac{dx}{1-2\sin x\cos x}$

24. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj $A = AA^T$, then find the value of $5a + b$.

25. Find the projection of the vector $2\hat{i}-3\hat{j}-6\hat{k}$ on vector joining the points (5, 6, – 3) and (3, 4, – 2).

OR

If $\vec{a} = 4\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{b} = 3\hat{i}+2\hat{k}$, find $|\vec{b}\times 2\vec{a}|$.

26. Suppose that two cards are drawn at random from a deck of 52 cards. Let X be the number of aces obtained. Then, find the probability distribution of X.

27. If $y = \sin^{-1}x$, then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

28. Find the solution of the differential equation $\frac{dy}{dx} = \frac{y(1+x)}{x(y-1)}$.

OR

Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y;\ y = 0, x = 0$.

29. Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$

30. Find the point on the parabola $y^2 = 2x$ which is closed to the point (1, 4).

31. If y = $e^{x\sin^2 x} + (\sin x)^x$, find $\frac{dy}{dx}$.

OR

Differentiate $\dfrac{2^{x^2+x+1}\cdot\sqrt{4x-1}}{(x^2-1)^{3/2}}$ w.r.t. x.

32. Let $f : A \to B$ be a function defined as $f(x)=\dfrac{2x+3}{x-3}$, where $A = R - \{3\}$ and $B = R - \{2\}$. Is the function f one-one and onto?

33. Find the area of the region bounded by the curve $y = x^2 + x$, the x-axis and the lines $x = 2$, $x = 5$.

34. Evaluate : $\int_0^1 \tan^{-1} x\, dx$

OR

Evaluate : $\int_1^2 \dfrac{dx}{x(1+x^2)}$

35. If $f(x)$ is continuous at $x = 0$, where

$$f(x)=\begin{cases}\dfrac{\sin x}{x}+\cos x, & \text{for } x>0\\ \dfrac{4(1-\sqrt{1-x})}{x}, & \text{for } x<0\end{cases}$$, then find $f(0)$.

Section-V

36. Find the vector and cartesian equation of the line through the point $\hat{i}+\hat{j}-3\hat{k}$ and perpendicular to the lines $\vec{r}=2\hat{i}-3\hat{j}+\lambda(2\hat{i}+\hat{j}-3\hat{k})$ and $\vec{r}=3\hat{i}-5\hat{j}+\mu(\hat{i}+\hat{j}+\hat{k})$.

OR

The four points $A(3, 2, -5)$, $B(-1, 4, -3)$, $C(-3, 8, -5)$ and $D(-3, 2, 1)$ are coplanar. Find the equation of the plane containing them.

37. Find the minimum value of $Z = 3x + 4y + 270$ subject to the constraints

$x + y \le 60$
$x + y \ge 30$
$x \le 40,\ y \le 40$
$x \ge 0,\ y \ge 0$

OR

Find the point for which the maximum value of $Z = x + y$ subject to the constraints $2x + 5y \le 100$, $\dfrac{x}{25}+\dfrac{y}{50}\le 1,\ x\ge 0,\ y\ge 0$ is obtained.

38. If $A=\begin{pmatrix}2 & 3 & 7\\ 3 & -2 & -1\\ 1 & 1 & 2\end{pmatrix}$, find A^{-1}. Using A^{-1} solve the following system of equations :

$2x + 3y + 7z = 12$
$3x - 2y - z = 0$
$x + y + 2z = 4$

OR

If $A=\begin{bmatrix}1 & -2 & 1\\ -2 & 3 & 1\\ 1 & 1 & 5\end{bmatrix}$, find (adj A^{-1}).

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