CODE:AG-11-TMC-TS-XII-1403 REG.NO:-TMC -D/79/89/36/63

General Instructions :-

(i) **All** Question are compulsory :

(ii) This question paper contains **36** questions.

(iii) Question **1-20** in **PART- A are** Objective type question carrying **1** mark each.

(iv) Question **21-26** in **PART -B** are sort-answer type question carrying **2** mark each.

(v) Question **27-32** in **PART -C** are long-answer-**I** type question carrying **4** mark each.

(vi) Question **33-36** in **PART -D** are long-answer-**II** type question carrying **6** mark each

(vii) You have to attempt only one if the alternatives in all such questions.

(viii)Use of calculator is not permitted.

(ix) Please check that this question paper contains 14 printed pages.

(x) Code number given on the right-hand side of the question paper should be written on the title page of the answer-book by the candidate.

Time : 3 Hours Maximum Marks : 80

CLASS – XII **MATHEMATICS**

PRE-BOARD EXAMINATION 2020-21

PART – A (**Question 1 to 20 carry 1 mark each.**)

SECTION I: Single correct answer type

This section contains 12 multiple choice question. Each question has four choices (A) , (B) , (C) &(D) out of which ONLY ONE is correct .

Suppose you visit to a hotel with your family and you observe that floor of a hotel is made up of mirror polished Kota stone. Also, there is a large crystal chandelier attached at the ceiling of the hotel. Consider the floor of the hotel as a plane having equation 3x – y + 4z = 2 and crystal chandelier as the point (3, –2, 1).

Based on the above information, answer the following questions:

Q.1	The d.r.'s of the perpendicular from the point (3, –2, 1) to the plane 3x – y + 4z = 2, is (a) < 3,1,4 > (b) < 3, -1,4 > (c) < 4,1,3 > (d) < 4,-1,3 > ans b
Q.2	The length of the perpendicular from the point (3, –2, 1) to the plane 3x – y + 4z = 2, is

	(a) $\sqrt{13}$ units (b) $\frac{1}{2}\sqrt{13}$ units (c) $\sqrt{\frac{13}{2}}$ units (d) $\frac{13}{\sqrt{2}}$ units ans c
Q.3	The equation of the perpendicular from the point (3, –2, 1) to the plane 3x – y + 4z = 2, is (a) $\frac{x-3}{3}=\frac{y-2}{-1}=\frac{z-1}{4}$ (b) $\frac{x-3}{3}=\frac{y+2}{-1}=\frac{z-1}{4}$ (c) $\frac{x+3}{3}=\frac{y+2}{-1}=\frac{z-1}{4}$ (d) None of these ans b
Q.4	The foot of the perpendicular drawn from the point (3, –2, 1) to the plane 3x – y + 4 z = 2, is (a) $\left(\frac{3}{2},\frac{-3}{2},-1\right)$ (b) $\left(\frac{-3}{2},\frac{3}{2},-1\right)$ (c) $\left(\frac{3}{2},\frac{3}{2},-1\right)$ (d) $\left(\frac{1}{2},\frac{3}{2},-1\right)$ ans a
Q.5	The image of the point (3, –2, 1) in the given plane is (a) (0, 1, 3) (b) (0, –1, 3) (c) (0, 1, –3) (d) (0, –1, –3) ans d
Q.6	If $\cos^{-1} p + \cos^{-1} q + \cos^{-1} r = \pi$ then $p^2 + q^2 + r^2 + 2pqr =$ (a)3 (b) 1 (c) 2 (d) – 1 (b) **Trick :** According to given condition, we

	put $p=q=r=\frac{1}{2}$. Then, $p^2+q^2+r^2+2pqr$ $=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+2.\frac{1}{2}.\frac{1}{2}.\frac{1}{2}$ $=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{2}{8}=1.$
Q.7	If the points whose position, vectors are $3\mathbf{i}-2\mathbf{j}-\mathbf{k}$, $2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $4\mathbf{i}+5\mathbf{j}+\lambda\mathbf{k}$ lie on a plane, then $\lambda=$ (a) $-\frac{146}{17}$ (b) $\frac{146}{17}$ (c) $-\frac{17}{146}$ (d) $\frac{17}{146}$ (a) Let $\mathbf{a}=3\mathbf{i}-2\mathbf{j}-\mathbf{k}$, $\mathbf{b}=2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$, $\mathbf{c}=-\mathbf{i}+\mathbf{j}+2\mathbf{k}$ and $\mathbf{d}=4\mathbf{i}+5\mathbf{j}+\lambda\mathbf{k}$. Since the points are coplanar, So, $[\mathbf{d}\ \mathbf{b}\ \mathbf{c}]+[\mathbf{d}\ \mathbf{c}\ \mathbf{a}]+[\mathbf{d}\ \mathbf{a}\ \mathbf{b}]=[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ $\Rightarrow\begin{vmatrix}4&5&\lambda\\2&3&-4\\-1&1&2\end{vmatrix}+\begin{vmatrix}4&5&\lambda\\-1&1&2\\3&-2&-1\end{vmatrix}+\begin{vmatrix}4&5&\lambda\\3&-2&-1\\2&3&-4\end{vmatrix}$

	$=\begin{vmatrix}3&-2&-1\\2&3&-4\\-1&1&2\end{vmatrix}$ $\Rightarrow 40+5\lambda+37-\lambda+94+13\lambda=25\Rightarrow\lambda=\frac{-146}{17}$.
Q.8	If $A=\begin{pmatrix}1&2&3\\3&1&2\\2&3&1\end{pmatrix}$ and $B=\begin{pmatrix}-5&7&1\\1&-5&7\\7&1&-5\end{pmatrix}$ then AB is equal to (a) I_3 (b) $2I_3$ (c) $4I_3$ (d) $18I_3$ (d) We have $A=\begin{bmatrix}1&2&3\\3&1&2\\2&3&1\end{bmatrix}$ and $B=\begin{bmatrix}-5&7&1\\1&-5&7\\7&1&-5\end{bmatrix}$ $\therefore AB=\begin{bmatrix}1&2&3\\3&1&2\\2&3&1\end{bmatrix}\begin{bmatrix}-5&7&1\\1&-5&7\\7&1&-5\end{bmatrix}$ $AB=\begin{bmatrix}18&0&0\\0&18&0\\0&0&18\end{bmatrix}=18\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $AB=18I_3$

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Q.9 Inverse of the matrix $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 & 5 \\ 7 & 4 & 6 \\ 4 & 2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & -4 \\ 8 & -4 & -5 \\ 3 & 5 & 2 \end{bmatrix}$ (c) Let

$A = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$, then $|A| = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix} = 1$ The

matrix of cofactors of $A = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$

Therefore, $adj(A) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}.adjA = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$, $(\because |A| = 1)$

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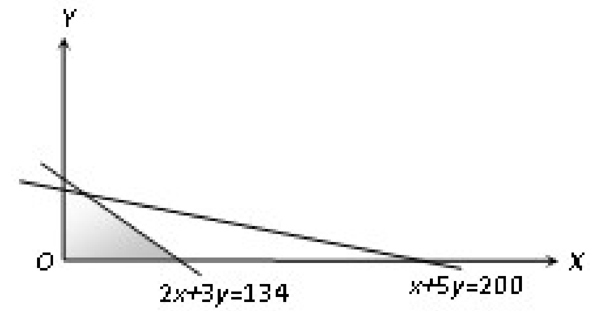
Q.10 The minimum value of objective function $c = 2x + 2y$ in the given feasible region, is



(a) 134 (b) 40 (c) 38 (d) 80 ans D

Fill in the blanks (Q11 – Q15)

Q.11 If $f : R_+ \to [4, \infty)$ & : $f(x) = x^2 + 4$ then $f^{-1}(x) =$------- $\sqrt{x-4}$

Q.12 he value of constant k = …….. so that the given function is continuous at the indicate point;

$f(x) = \begin{cases} \frac{1 - \cos 2kx}{x^2}, & if\ x \neq 0 \\ 8 & if\ x = 0 \end{cases}$ at x = 0 ANS. $k = \pm 2$

Q.13 If $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$, then x = …… .Ans; $x = -2$

Q.14 Which of the following is not a decreasing function on

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	the interval $\left(0,\frac{\pi}{2}\right)$ (a) $\cos x$ (b) $\cos 2x$ (c) $\cos 3x$ (d) $\cot x$ ANS " C OR The function $f(x)=2x^3-15x^2+36x+4$ is maximum at (a) $x=2$ (b) $x=4$ (c) $x=0$ (d) $x=3$ ANS " A
Q.15	In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i}-\hat{j}+2\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$ respectively. Find the vector representing CA . Ans : -(3i+2j+7k) **OR** If $\lvert\vec{a}\times\vec{b}\rvert=4$, $\lvert\vec{a}.\vec{b}\rvert=2$, then $\lvert\vec{a}\rvert^2\lvert\vec{b}\rvert^2=$ ___ . Ans. =20

(Q16 - Q20) Answer the following questions

Q.16	Find the real values of λ for which the following system of linear equations has non-trivial solutions. $2\lambda x-2y+3z=0$; $x+\lambda y+2z=0$; $2x+\lambda z=0$ Ans; $\lambda=2$,
Q.17	Evaluate: $\int_0^2 x\sqrt{(2-x)}dx.$ ANS : $\frac{16\sqrt{2}}{15}$

Q.18	Evaluate: $\int \frac{x^3}{\sqrt{x^2+2}}dx=$ (a) $\frac{1}{3}(x^2+2)^{3/2}+2(x^2+2)^{1/2}+c$ (b) $\frac{1}{3}(x^2+2)^{3/2}-2(x^2+2)^{1/2}+c$ (c) $\frac{1}{3}(x^2+2)^{3/2}+(x^2+2)^{1/2}+c$ 7 (d) $\frac{1}{3}(x^2+2)^{3/2}-(x^2+2)^{1/2}+c$ (b) $\int\frac{x^3}{\sqrt{x^2+2}}dx=\int\frac{x^2.x}{\sqrt{x^2+2}}dx$ Put $x^2+2=t^2 \Rightarrow x\,dx=t\,dt$ and $x^2=t^2-2$, then it reduces to $\int\frac{(t^2-2)}{t}t\,dt=\int(t^2-2)dt$ $=\frac{t^3}{3}-2t+c=\frac{(x^2+2)^{3/2}}{3}-2(x^2+2)^{1/2}+c.$
Q.19	Evaluate: $\int\frac{dx}{x\log x\log(\log x)}$ $\int\frac{dx}{x\log x.\log(\log x)}$ Put $\log x=t,$ then it reduces to $\int\frac{dt}{t.\log(t)}$ Again put $\log t=z,$ then reduces form is $\int\frac{dz}{z}=\log z=\log[\log(\log x)]+c$. **OR**

Evaluate: $\int \tan^4 x\, dx$ $\int \tan^4 x\,dx = \int \tan^2 x(\sec^2 x - 1)\,dx$

$$= \int \tan^2 x \sec^2 x\,dx - \int \tan^2 x\,dx = \frac{\tan^3 x}{3} - \tan x + x + c.$$

Q.20 For what value of a the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear .

Let $\vec{p} = 2\hat{i} - 3\hat{j} + 4k$ and $\vec{q} = a\hat{i} + 6\hat{j} - 8k$

Two vectors $\vec{p}$ and $\vec{q}$ will be collinear if,

$$\vec{p} \times \vec{q} = 0 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & k \\ 2 & -3 & 4 \\ a & 6 & -8 \end{vmatrix} = 0$$

$\Rightarrow \hat{i}(24 - 24) - \hat{j}(-16 - 4a) + k(12 + 3a) = 0$

$\Rightarrow 0\hat{i} + (16 + 4a)\hat{j} + (12 + 3a)k = 0\hat{i} + 0\hat{j} + 0k$

$\Rightarrow 16 + 4a = 0$ and $12 + 3a = 0$

$\Rightarrow a = -4$

PART – B **(Question 21 to 26 carry 2 mark each.)**

Q.21 Prove that : $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

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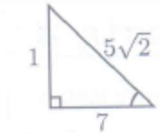
Solution: L.H.S. $= 2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8}$



$= \left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{5}\right) + \tan^{-1}\frac{1}{7} + \left(\tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{8}\right)$

$= \tan^{-1}\left[\dfrac{\frac{1}{5} + \frac{1}{5}}{1 - \frac{1}{5} \times \frac{1}{5}}\right] + \tan^{-1}\frac{1}{7} + \tan^{-1}\left[\dfrac{\frac{1}{8} + \frac{1}{8}}{1 - \frac{1}{8} \times \frac{1}{8}}\right]$ $\left[\because \left(\frac{1}{5}\right)\left(\frac{1}{5}\right) < 1 \text{ and } \left(\frac{1}{8}\right)\left(\frac{1}{8}\right) < 1\right]$

$= \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{16}{63}$

$= \tan^{-1}\left[\dfrac{\frac{5}{12} + \frac{1}{7}}{1 - \frac{5}{12} \times \frac{1}{7}}\right] + \tan^{-1}\frac{16}{63}$ $\left[\because \left(\frac{5}{12}\right)\left(\frac{1}{7}\right) < 1\right]$

$= \tan^{-1}\frac{47}{79} + \tan^{-1}\frac{16}{63}$

$= \tan^{-1}\left[\dfrac{\frac{47}{79} + \frac{16}{63}}{1 - \frac{47}{79} \times \frac{16}{63}}\right]$ $\left[\because \left(\frac{47}{79}\right)\left(\frac{16}{63}\right) < 1\right]$

$= \tan^{-1}1 = \frac{\pi}{4} =$ R.H.S.

OR

Relation R in the set A = {1, 2, 3, 4, 5,6,7} given by R = {(a, b): $|a - b|$ is even} Then find the number of set of all elements to related to 3 .4

Q.22 If $y = \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$, find $\frac{dy}{dx}$.

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	(iii) Let $y = \sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$. Putting $x = \cos\theta$, we get $y = \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right] = \sin\left[2\tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right]$ $= \sin\left[2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right]$ $= \sin\theta$ $= \sqrt{1-\cos^2\theta} = \sqrt{1-x^2}$. Differentiating both sides w.r.t. x, we get $\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{1-x^2}\right) = \frac{\frac{d}{dx}(1-x^2)}{2\sqrt{1-x^2}} = -\frac{x}{\sqrt{1-x^2}}$.
Q.23	Find the equation of a tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ at a point where $t = \frac{\pi}{2}$. Here, $x = a\sin^3 t$, $y = b\cos^3 t$ Differentiating (1) wrt t $\frac{dx}{dt} = 3a\sin^2 t \times \cos t$ and $\frac{dy}{dt} = -3b\cos^2 t \times \sin t$

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	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3b\cos^2 t \times \sin t}{3a\sin^2 t \times \cos t} = -\frac{b}{a}\cot t$ $\therefore$ Slope of the tangent at $t = \frac{\pi}{2}$ $\left.\frac{dy}{dx}\right]_{\frac{\pi}{2}} = -\frac{b}{a}\cot\frac{\pi}{2} = 0$ Hence, equation of tangent is given by $y - b\cos^3\frac{\pi}{2} = 0$ or $y = 0$
Q.24	If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of the pints A,B,C and D respectively, find the angle between $\vec{AB}$ and $\vec{CD}$. Deduce that $\vec{AB}$ and $\vec{CD}$ are parallel . Ans. $\theta = \pi$ & $\vec{AB} = \lambda\vec{CD}$ **OR** Find the values of 'a' for which the vector $\vec{r} = (a^2-4)i + 2j - (a^2-9)k$ makes acute angles with the

coordinate axes. Ans : $\vec{r}.i > 0, \vec{r}.j > 0$ and $\vec{r},k > 0$

$\vec{F}.i > 0$ and $\vec{r}.k > 0$

$(a^2-4) > 0$ and $-(a^2-9) > 0$

$(a-2)(a+2) >$ and $(a+3)(a-3) < 0$

$a \in (-3,-2) \cup (2,3)$

Q.25 Show that the line $\vec{r} = (\hat{i}+\hat{j}) + \lambda(2\hat{i}+\hat{j}+4\hat{k})$ is parallel to the plane $\vec{r}.(-2\hat{i}+\hat{k}) = 5$ .Also find the distance between the line and the plane. Ans Required Condition for line // to plane is $\vec{b} \bullet \vec{n} = 0$ and distance between plane and line $\frac{7}{\sqrt{5}}$

Q.26 Two cards are drawn without replacement from a well shuffled pack of 52 cards. Find the probability that one is a king and other is a queen of opposite color. Ans Required Probability = $= \frac{4}{52} \times \frac{2}{51} + \frac{4}{52} \times \frac{2}{51} = \frac{4}{663}$

PART – C (Question 27 to 32 carry 4 mark each.)

Q.27 If $f : R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ & $g : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$. Prove that

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$gof = I_A$ & $(fog) = I_B$ where $B = R - \left\{\frac{3}{5}\right\}$ & $A = R - \left\{\frac{7}{5}\right\}$.

Find also g^{-1}, f^{-1} & $(gof)^{-1}$. Ans : $g^{-1} = \frac{3x+4}{5x-7}$, $f^{-1} = \frac{7x+4}{5x-3}$ & $(gof) = (gof)^{-1} = x$

Q.28 If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$.

$\Rightarrow y' = \cos(\sin x)\cos x \Rightarrow \sec x.y' = \cos(\sin x)$

again differentiating w.r.t x, we can get

$\sec x\, y'' + y' \sec x \tan x = -\sin(\sin x)\cos x$

$\Rightarrow \tan x\ y' = -y\ .\ \cos^2 x \Rightarrow y'' + (\tan x)\ y' + y\cos^2 x = 0$

OR

Find the derivative of the $\cos^{-1}\left(\sin\sqrt{\frac{1+x}{2}}\right) + x^x$ f(x) w.r.t. x at x = 1.

Ans. Let $y = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} + x^x$

$\therefore \quad y = u + v$

Let $u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\}$; $v = x^x$ $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$$u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} = \cos^{-1}\left[\cos\cdot\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right]$$

$$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}} \quad \therefore \frac{du}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} \text{ .... (i)}$$

$v = x^x$

$\therefore \log v = x \log x$ $\frac{1}{v}\frac{dv}{dx} = x \cdot \frac{1}{x} + 1 \log x = 1 + \log x$

$$\frac{dv}{dx} = x^x (1 + \log x) \text{ ....(ii)}$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} + x^x (1 + \log x)$$

$$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = -\frac{1}{4} + 1 = \frac{3}{4}$$

Q.29 Find the particular solution of the differential equation $\frac{dy}{dx} + y \tan x = 3x^2 + x^3 \tan x, x \neq \frac{\pi}{2}$, given that $y = 0$ when $x = \frac{\pi}{3}$.

This is linear differential equation of the form $\frac{dy}{dx} + yP(x) = Q(x)$, where $P(x) = \tan x$ and $Q(x) = 3x^2 + x^3 \tan x$.

Now Integrating Factor, I.F. $= e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

So the solution is given by : $y \times \sec x = \int \sec x \times \left[3x^2 + x^3 \tan x\right] dx + C$

$\Rightarrow y \sec x = \int\left[3x^2 \sec x + x^3 \sec x \tan x\right] dx + C$

$\Rightarrow y \sec x = \int 3x^2 \sec x \, dx + \int x^3 \sec x \tan x dx + C$

$\Rightarrow y \sec x = \int 3x^2 \sec x \, dx + x^3 \int \sec x \tan x dx - \int\left(\frac{d}{dx}x^3 \int \sec x \tan x dx\right) dx + C$

$\Rightarrow y \sec x = \int 3x^2 \sec x \, dx + x^3 \sec x - \int 3x^2 \sec x \, dx + C$

$\Rightarrow y \sec x = x^3 \sec x + C$

Since $x = \frac{\pi}{3}$ when $y = 0$ so, $\Rightarrow 0 \sec\frac{\pi}{3} = \left(\frac{\pi}{3}\right)^3 \sec\frac{\pi}{3} + C \quad \Rightarrow C = -\frac{2\pi^3}{27}$

$\therefore y \sec x = x^3 \sec x - \frac{2\pi^3}{27}$ or $27y = 27x^3 - 2\pi^3 \cos x$ is the required solution.

Q.30 Evaluate: $\int \frac{(3\sin\alpha - 2)\cos\alpha}{5 - \cos^2\alpha - 4\sin\alpha} d\alpha$.

Evaluate: $\int \frac{(3\sin\alpha - 2)\cos\alpha}{5 - \cos^2\alpha - 4\sin\alpha} d\alpha$

Let $I = \int \frac{(3\sin\alpha - 2)\cos\alpha}{5 - \cos^2\alpha - 4\sin\alpha} d\alpha$

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	Let $t=\sin\alpha \Rightarrow dt=\cos\alpha\, d\alpha$ $\Rightarrow I=\int \frac{(3t-2)}{5-(1-t^2)-4t}dt$ $\Rightarrow I=\int \frac{(3t-2)}{t^2-4t+4}dt=\int \frac{(3t-2)}{(t-2)^2}dt$ Using the method of Partial fractions $\frac{(3t-2)}{(t-2)^2}=\frac{A}{(t-2)}+\frac{B}{(t-2)^2} \Rightarrow A=3, B=4$ $I=\int\left[\frac{3}{(t-2)}+\frac{4}{(t-2)^2}\right]dt$ $\Rightarrow I=\int \frac{3}{(t-2)}dt+\int \frac{4}{(t-2)^2}dt$ $=3\log\lvert\sin\alpha-2\rvert-\frac{4}{\sin\alpha-2}+C$ **OR** Evaluate: $\int \cos 2\theta \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)d\theta$ .We know that $\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)=\log\left(\frac{1+\tan\theta}{1-\tan\theta}\right)=\log\tan\left(\frac{\pi}{4}+\theta\right)$ $\int \sec\theta\, d\theta=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ $\therefore \int \sec 2\theta\, d\theta=\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)$

	$\therefore\ 2\sec 2\theta=\frac{d}{d\theta}\log\tan\left(\frac{\pi}{4}+\theta\right)$ .....(i) Integrating the given expression by parts, we get $I=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\int \sin 2\theta\,.\,2\sec 2\theta\, d\theta$ by (i) $=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\int \tan 2\theta\, d\theta$ $=\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta$.
Q.31	The members of a consulting firm rent cars from three rental agencies:50% from agency X, 30% from agency Y and 20% from agency Z. From past experience it is known that 9% of the cars from agency X need a service and tuning before renting, 12% of the cars from agency Y need a service and tuning before renting and 10% of the cars from agency Z need a service and tuning before renting. If the rental car delivered to the firm needs service and tuning, find the probability that agency Z is not to be blamed. Let A be the event that car delivered to firm needs service and tuning. Also let E_1, E_2 and E3 be the events of car being rented from agencies X, Y and Z respectively.

$P(E_1)=\frac{50}{100}$ $P(E_2)=\frac{30}{100}$ $P(E_3)=\frac{20}{100}$

$P(A|E_1)=\frac{9}{100}$ $P(A|E_2)=\frac{12}{100}$ $P(A|E_3)=\frac{10}{100}$

$$P(E_3|A)=\frac{P(E_3)P(A|E_3)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$P(E_3|A)=\frac{\frac{20}{100}\times\frac{10}{100}}{\frac{50}{100}\times\frac{9}{100}+\frac{30}{100}\times\frac{12}{100}+\frac{20}{100}\times\frac{10}{100}}=\frac{20}{101}$$

$$P(E'_3|A)=1-P(E_3|A)=1-\frac{20}{101}=\frac{81}{101}$$

OR

Bag I contains 4 red and 5 black balls and bag II contains 3 red and 4 black balls. One ball is transferred from bag I to bag II and then two balls are drawn at random (without replacement) from bag II. The balls so drawn are both found to be black. Find the probability that the transferred ball is black.

Let E_1 : Event that transferred ball is black

E_2 : Event that transferred ball is Red

E_3 : Event that balls drawn are black $P(E_1)=\frac{5}{9}$, $P(E_2)=\frac{4}{9}$

$P(A/E_1)=\frac{^5C_2}{^8C_2}=\frac{5}{14}$, $P(A/E_2)=\frac{^4C_2}{^8C_2}=\frac{3}{14}$

$$P(E_1/A)=\frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\ P(A/E_1)+P(E_2)\ P(A/E_2)}$$

$$=\frac{\frac{5}{9}\times\frac{5}{14}}{\frac{5}{9}\times\frac{5}{14}+\frac{4}{9}\times\frac{3}{14}}=\frac{25}{37}$$

Q.32 A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit, of Rs 17.50 per package on nuts and Rs. 7.00 per package on bolts. How many packages of each should e produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?

Answer : Let the manufacturer produce x packages of nuts and y packages of bolts. Therefore, $x \ge 0$ and $y \ge 0$

The given information can be compiled in a table as follows.

	Nutes	Bolt	Availability
Machine A(h)	1	3	12
Machine B(h)	3	1	12

The profit on a package of nuts is Rs 17.50 and on a package of bolts is Rs 7. Therefore, the constraints are

$x + 3y \le 12$

$3x + y \le 12$

Total profit, $Z = 17.5x + 7y$

The mathematical formulation of the given problem is

Maximise $Z = 17.5x + 7y$

subject to the constraints,

$x + 3y \le 12$

$3x + y \le 12$

$x, y \ge 0$

The feasible region determined by the system of

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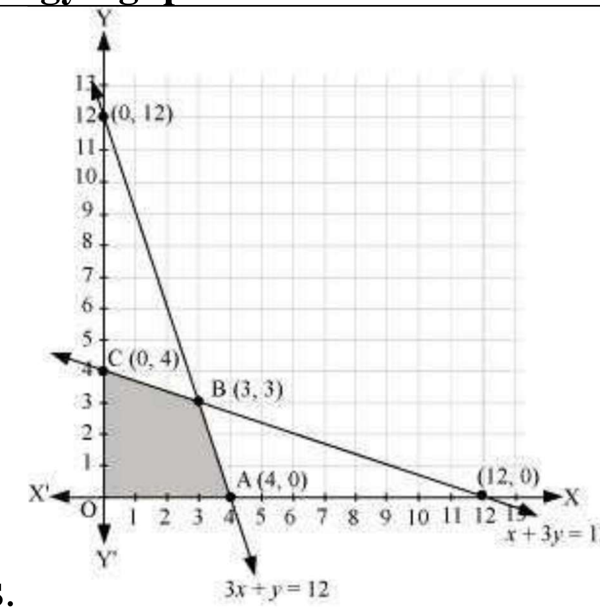

constraints is as follows.

The corner points are A(4, 0), B(3, 3), and C(0, 4).

The values of Z at these corner points are as follows.

Corner point	$z = 17.5x + 7y$	
O(0,0)	0	
A(4,0)	70	
B(3,3)	73.5	→ Maximum
C(0,4)	28	

The maximum value of Z is Rs 73.50 at (3, 3).

Thus, 3 packages of nuts and 3 packages of bolts should be produced each day to get the maximum profit of Rs 73.50.

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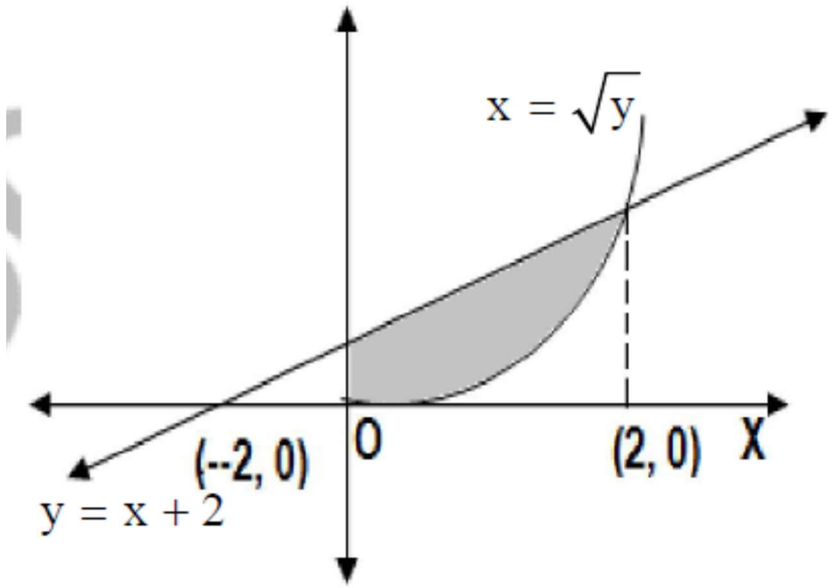
PART – D (Question 33 to 36 carry 6 mark each.)

Q.33 Using integration, find the area of the region bounded by the line $x - y + 2 = 0$ the curve $x = \sqrt{y}$ and y-axis.



We have $x - y + 2 = 0 \ldots$(i) and $x = \sqrt{y} \ldots$(ii)

Solving (i) & (ii), $x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0 \quad \therefore x = 2, \ \because x = -1$

Required area $= \int_0^2 \left[y_{(i)} - y_{(ii)} \right] dx$

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$= \int_0^2 \left[x + 2 - x^2 \right] dx$

$= \left[\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3 \right]_0^2$

$= \left[2 + 4 - \frac{8}{3} - 0 \right] = \frac{10}{3}$ sq.units.

Q.34 If $\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then using properties of determinants, find the value of $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$, where $x, y, z \neq 0$

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Taking x, y, z common from C_1, C_2, C_3 respectively, we get

$$xyz\begin{vmatrix} a/x & b/y-1 & c/z-1 \\ a/x-1 & b/y & c/z-1 \\ a/x-1 & b/y-1 & c/z \end{vmatrix}=0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a/x+b/y+c/z-2 & b/y-1 & c/z-1 \\ a/x+b/y+c/z-2 & b/y & c/z-1 \\ a/x+b/y+c/z-2 & b/y-1 & c/z \end{vmatrix}=0$$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 1 & b/y & c/z-1 \\ 1 & b/y-1 & c/z \end{vmatrix}=0$$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\begin{vmatrix} 1 & b/y-1 & c/z-1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}=0$$

$$\therefore \quad \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}-2\right)\cdot 1=0 \Rightarrow \frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$

OR

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Given two matrix , $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ & $B=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$,

Verify that BA = 6I , use the result to solve the system x – y = 3 , 2x + 3y + 4z = 17 , y + 2z = 7 .

$$B=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \qquad A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$BA=\begin{bmatrix} 2\times1+2\times2-4\times0 & 2\times(-1)+2\times3-4\times1 & 2\times0+2\times4-4\times2 \\ -4+4 & 4+6-4 & 8-8 \\ 2-2 & -2-3+5 & -4+10 \end{bmatrix}$$

$$BA=\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}=6I$$

System of equations

x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7, can be written as

$$A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \qquad X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad C=\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$AX=C$

$BA = 6I \quad \Rightarrow \quad B = 6I\,A^{-1} \quad \Rightarrow A^{-1} = \frac{1}{6}B$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix}$$

11

$$X = \begin{bmatrix} \frac{12}{6} \\ \frac{-6}{6} \\ \frac{24}{6} \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

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$x = 2, y = -1, z = 4$

Q.35 Find the equation of tangents to the curve $y = \cos(x + y)$, $-2\pi \langle x \langle 2\pi$ that are parallel to the line $x + 2y = 0$.

$x + y = \frac{\pi}{2}, \frac{-3\pi}{2} \Rightarrow y = 0 \therefore \left(\frac{\pi}{2}, 0\right)\left(\frac{-3\pi}{2}, 0\right) \Rightarrow x + 2y = \frac{\pi}{2}, \frac{-3\pi}{2}$

OR

A large window is in the form of a rectangle surmounted by a Equilateral triangle . The total perimeter of the window is 12 m, find the dimensions of the window to admit maximum light through the whole opening .

Solution: Let A denote the area of a window consisting of a rectangle of length x and 12 adth y surmounted by an equilateral triangle of side x.

$$\Rightarrow \quad A = xy + \frac{\sqrt{3}}{4}x^2 \quad \text{...(1)}$$

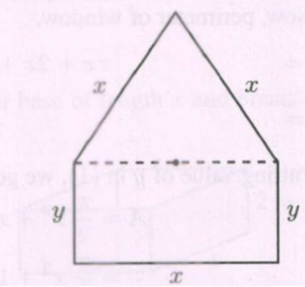
Now, perimeter of window, $P = 3x + 2y$.

Also, given that $P = 12$.

$$\Rightarrow \quad 12 = 3x + 2y$$

$$\Rightarrow \quad y = 6 - \frac{3}{2}x \quad \text{...(2)}$$

Putting value of y in (1), we get



$$A = x\left(6 - \frac{3}{2}x\right) + \frac{\sqrt{3}}{4}x^2$$

$$= 6x - x^2\left(\frac{3}{2} - \frac{\sqrt{3}}{4}\right)$$

Then, $\frac{dA}{dx} = 6 - 2x\left(\frac{3}{2} - \frac{\sqrt{3}}{4}\right)$

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Now, $\frac{dA}{dx} = 0 \Rightarrow 6 - 2x\left(\frac{3}{2} - \frac{\sqrt{3}}{4}\right) = 0$

$\Rightarrow x = \frac{12}{6-\sqrt{3}}$

Also, $\frac{d^2A}{dx^2} = -2\left(\frac{3}{2} - \frac{\sqrt{3}}{4}\right) = -\frac{1}{2}(6-\sqrt{3})$

$\Rightarrow \left(\frac{d^2A}{dx^2}\right)_{x=12/(6-\sqrt{3})} = -\frac{1}{2}(6-\sqrt{3}) < 0$

$\therefore$ By second derivative test, A is maximum when $x = \frac{12}{6-\sqrt{3}}$.

Putting value of x in (2), we get

$$y = 6 - \frac{18}{6-\sqrt{3}} = \frac{18-6\sqrt{3}}{6-\sqrt{3}}.$$

Hence, length of rectangle $= \frac{12}{6-\sqrt{3}}$ m and breadth of rectangle $= \frac{18-6\sqrt{3}}{6-\sqrt{3}}$ m.

Q.36 Find the distance of the point (3 , -2, 1) from the plane $3x + y - z + 2 = 0$ measured parallel to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also find the foot of the perpendicular from the given point upon the given line .

The equation of the line passing through the point(3, -2, 1) and parallel to the given line is

$$\frac{x-3}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$$

Any point on this line is $(2\lambda+3, -3\lambda-2, \lambda+1)$

If it lies on the plane, we have $3(2\lambda+3) - 3\lambda - 2 - \lambda - 1 + 2 = 0 \Rightarrow \lambda = -4$

Hence, the point common to the plane and the line is (-5, 10, -3).

Hence, the required distance $= \sqrt{(3+5)^2 + (-2-10)^2 + (1+3)^2}$ units $= 4\sqrt{14}$ units

The equation of the line passing through (3, -2, 1) and perpendicular to the plane is

$$\frac{x-3}{3} = \frac{y+2}{1} = \frac{z-1}{-1}$$

Any point on it is $(3\mu+3, \mu-2, -\mu+1)$

If it lies on the plane, we get $3(3\mu+3) + \mu - 2 + \mu - 1 + 2 = 0 \Rightarrow \mu = \frac{-8}{7}$

The required foot of the perpendicular $= \left(\frac{-3}{7}, \frac{-22}{7}, \frac{15}{7}\right)$

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