Subject Code : **041**

SQP-8

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

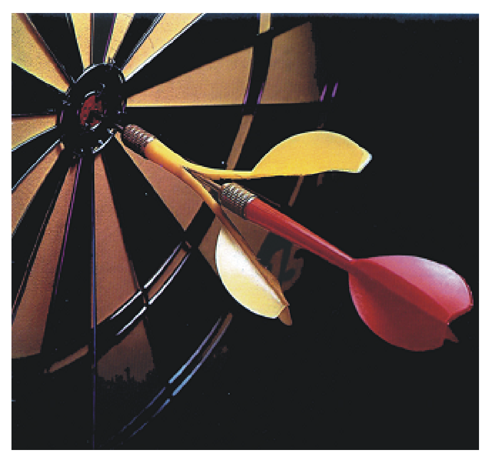
1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*



PART - A

Section - I

1. Evaluate : $\int \frac{(a^x + b^x)^2}{a^x b^x} dx$

OR

Evaluate : $\int \frac{dx}{\sqrt{1-2x-x^2}}$

2. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then find the value of k.

3. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

OR

If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B \mid A) = 0.6$, then find $P(A \cup B)$.

4. Differentiate the function $\left(\frac{2\tan x}{\tan x + \cos x}\right)^2$ w.r.t. x.

5. Find the cofactors of the element of third row and second column of the following determinant $\begin{vmatrix} 1 & y & z+x \\ 1 & z & x+y \end{vmatrix}$.

OR

If A is a matrix of order 3×3 and $|A| = 5$, then find the value of $|\text{adj } A|$.

6. Set A has three elements and set B has four elements. Find the number of injections that can be defined from A to B.

7. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

OR

Find the solution of $y' = y \cot 2x$.

8. Find the principal value of $\cot^{-1}(-\sqrt{3})$.

9. Find the direction cosines of a line, for which $\alpha = \beta$ and $\gamma = 45°$.

OR

If $P = (-2, 3, 6)$, then find the d.c.'s of OP.

10. How many equivalence relations on the set $\{1, 2, 3\}$ containing $(1, 2)$ and $(2, 1)$ are there in all?

11. If the plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis, then find the value of α.

12. If A and B are two independent events such that $P(A \cup B) = 0.6$ and $P(A) = 0.2$, then find $P(B)$.

13. If $\begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$, then $x =$ ______.

14. If A and B are events such that $P(A) > 0$ and $P(B) \neq 1$, then prove that $P(A' \mid B') = \frac{1 - P(A \cup B)}{P(B')}$.

15. Find the value of k in the following probability distribution.

$X = x$	0.5	1	1.5	2
$P(X = x)$	k	k^2	$2k^2$	k

16. If the angle between $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j} + a\hat{k}$ is $\frac{\pi}{3}$, then find the value of a.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. A poster is to be formed for a company advertisement. The top and bottom margins of poster should be 4 cm and the side margins should be 6 cm. Also, the area for printing the advertisement should be 384 cm^2. Based on the above answer the following :

(i) If a be the width and b be the height of poster, then the area of poster, expressed in terms of a and b, is given by

(a) $288 + 8a + 12b$ (b) $8a + 12b$ (c) $384 + 8a + 12b$ (d) none of these

(ii) The relation between a and b is given by

(a) $a = \frac{288 + 12b}{b - 8}$ (b) $a = \frac{12b}{b - 8}$ (c) $a = \frac{12b}{b + 8}$ (d) none of these

(iii) Area of poster in terms of b is

(a) $\frac{12b^2}{b-8}$ (b) $\frac{288b+12b^2}{b-8}$ (c) $\frac{288b+12b^2}{b+8}$ (d) $\frac{12b^2}{b+8}$

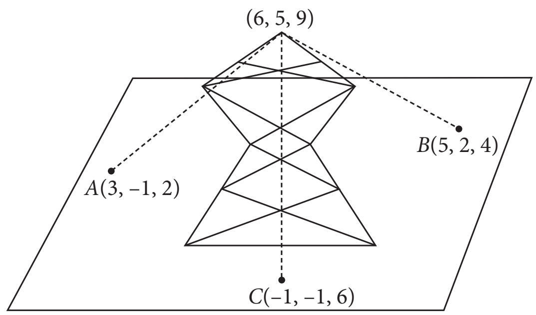
(iv) The value of b, so that area of the poster is minimized, is

(a) 24 (b) 36 (c) 18 (d) 22

(v) The value of a, so that area of the poster is minimized, is

(a) 24 (b) 36 (c) 18 (d) 22

18. Consider the earth as a plane having points $A(3, -1, 2)$, $B(5, 2, 4)$ and $C(-1, -1, 6)$ on it. A mobile tower is tied with 3 cables from the point A, B and C such that it stand vertically on the ground. The peak of the tower is at the point (6, 5, 9), as shown in the figure.



Based on the above answer the following :

(i) The equation of plane passing through the points A, B and C is

(a) $3x - 4y + 3z = 0$ (b) $3x - 4y + 3z = 19$ (c) $4x - 3y + 3z = 0$ (d) $4x - 3y + 3z = 19$

(ii) The height of the tower from the ground is

(a) 6 units (b) 5 units (c) $\frac{6}{\sqrt{34}}$ units (d) $\frac{5}{\sqrt{34}}$ units

(iii) The equation of line of perpendicular drawn from its peak to the ground is

(a) $\frac{x-6}{3} = \frac{y-4}{-5} = \frac{z-9}{3}$ (b) $\frac{x-6}{3} = \frac{y-5}{-4} = \frac{z-9}{3}$

(c) $\frac{x-6}{3} = \frac{y-4}{5} = \frac{z-9}{3}$ (d) $\frac{x-6}{3} = \frac{y-5}{4} = \frac{z-9}{3}$

(iv) The coordinates of foot of perpendicular are

(a) $\left(\frac{93}{17}, \frac{97}{17}, \frac{144}{17}\right)$ (b) $\left(\frac{144}{17}, \frac{97}{17}, \frac{93}{17}\right)$ (c) $\left(\frac{91}{17}, \frac{93}{17}, \frac{144}{17}\right)$ (d) none of these

(v) The area of ΔABC is

(a) $\sqrt{34}$ sq. units (b) $2\sqrt{34}$ sq. units (c) $\sqrt{17}$ sq. units (d) $2\sqrt{7}$ sq. units

PART – B

Section III

19. Find the derivative of the function $\sqrt{a + \sqrt{a + x}}$ w.r.t. x.

20. Evaluate : $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$

OR

Evaluate : $\int \frac{1}{\sin x + \sqrt{3}\cos x} dx$

21. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

Determine:

(i) K (ii) $P(X < 3)$

22. If $\sin [\cot^{-1} (x + 1)] = \cos (\tan^{-1} x)$, then find x.

23. Solve the differential equation $\cos^2(x-2y)=1-2\dfrac{dy}{dx}$.

OR

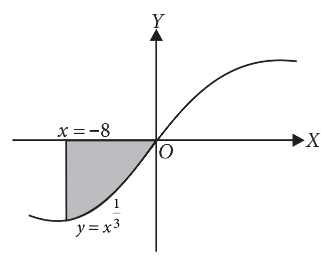
Find the solution of the differential equation $x+y\dfrac{dy}{dx}=\sec(x^2+y^2)$.

24. Find the equation of normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ at (1, 1).

25. If $P(\text{not } A) = 0.7$, $P(B) = 0.7$ and $P(B \mid A) = 0.5$, then find $P(A \mid B)$ and $P(A \cup B)$.

26. Find the inverse of the matrix $A=\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$.

27. Compute the shaded area shown in the given figure.



28. Find $|\vec{a}\times\vec{b}|$, if $\vec{a}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{b}=-\hat{i}+3\hat{k}$.

OR

Find the angle between two vectors $\vec{a}$ and $\vec{b}$ having the same length $\sqrt{2}$ and their scalar product is –1.

Section - IV

29. Let a relation R on the set A of real numbers be defined as $(a, b) \in R \Rightarrow 1 + ab > 0$ for all $a, b \in A$. Show that R is reflexive and symmetric but not transitive.

30. Sketch the graph $y = |x + 1|$. Evaluate $\int_{-4}^{2} |x + 1|\, dx$.

31. Evaluate : $\int \dfrac{x^2+9}{x^4+81}\, dx$

OR

Evaluate : $\int x^2 \sin 2x \, dx$

32. Solve : $\sin^{-1}\left(\dfrac{dy}{dx}\right)=x+y$

33. If $f(x)=\begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}$, then show that the function is discontinuous at $x = 0$.

34. If $(ax + b)\, e^{y/x} = x$, then show that $x^3 \frac{d^2 y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^2$.

OR

Find $\frac{dy}{dx}$, when $x = a\left\{\cos t + \frac{1}{2} \log \tan^2 \frac{t}{2}\right\}$ and $y = a \sin t$.

35. Find the equation of normal to the curve $16x^2 + 9y^2 = 144$ at $(2, y_1)$ where $y_1 > 0$.

Section-V

36. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{bmatrix}$, then find A^{-1}. Hence find $|\text{adj } A|$ and $|A^{-1}|$.

OR

Find the inverse of $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$. Hence find $(A^{-1})^2$.

37. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3$ and $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) + 11 = 0$ and passing through the point $(-2, 1, 3)$.

OR

Find the co-ordinates of the points on the line $x - 2 = \frac{y+3}{-2} = \frac{z+5}{2}$, which are on either side of the point $A(2, -3, -5)$ at a distance of 3 units from it.

38. Solve the following LPP graphically :

Maximize $Z = 600x + 400y$

subject to the constraints :

$x + 2y \leq 12,\ 2x + y \leq 12$

$x + \frac{5}{4} y \geq 5$ and $x, y \geq 0$.

OR

Find the number of points at which the objective function $z = 3x + 2y$ can be maximized subject to $3x + 5y \leq 15,\ 5x + 2y \leq 20,\ x \geq 0,\ y \geq 0$.

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