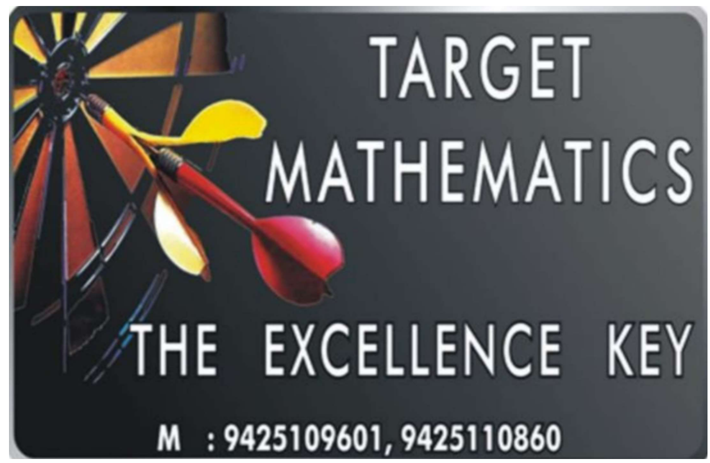
Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*



Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. If a matrix A is both symmetric and skew-symmetric, then show that A is a zero matrix.

OR

If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}=\begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then find the value of k.

2. Evaluate : $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\,dx$

3. If the direction ratios of a line are 1, –3, 2, then find its direction cosines.

OR

The coordinates of a point P are (3, 12, 4) w.r.t. origin O, then find the direction cosines of OP.

4. Let R be a relation defined on the set of natural numbers N as follow :
$R = \{(x, y) \mid x \in N, y \in N \text{ and } 2x + y = 24\}$
Find the domain and range of the relation R.

5. Find the distance from the origin to the plane $x + 3y - 2z + 1 = 0$.

OR

Find the foot of the perpendicular from (0, 0, 0) to $3x + 4y - 6z = 0$.

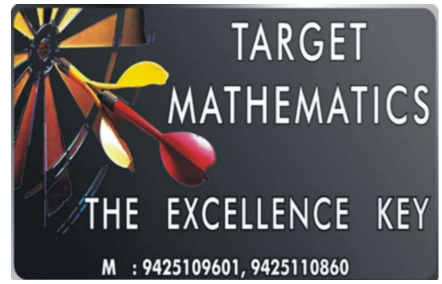
6. Construct a matrix $A = [a_{ij}]_{2\times2}$, where $a_{ij} = i + j$.

7. If $\vec{p} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are the position vectors of points P, Q respectively and point $R(\vec{r})$ divides the line PQ internally in the ratio 2 : 1, then find the coordinates of R.

OR

If $\vec{a} = \hat{i} + 3\hat{j}$, $\vec{b} = 2\hat{i} + 5\hat{j}$, $\vec{c} = 4\hat{i} + 2\hat{j}$ and $\vec{c} = t_1\vec{a} + t_2\vec{b}$, then find the value of t_1 and t_2.

8. Find the principal value of $\tan^{-1}(-\sqrt{3})$.



9. Evaluate : $\int_0^{\pi/4} \tan x \, dx$

OR

Evaluate : $\int x \cot^{-1} x \, dx$

10. Find the value of $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{i}\times\hat{k}) + \hat{k}\cdot(\hat{i}\times\hat{j})$.

11. If (2, 4, –3) is the foot of the perpendicular drawn from the origin to a plane, then find the equation of the plane.

12. Find the domain of $f(x) = \sin^{-1} x + \tan^{-1} x + \sec^{-1} x$.

13. Find the equation of the line in symmetric form which passes through the points $A(-2, -1, 5)$ and $B(1, 3, -1)$.

14. Find the value of x for which the matrix $A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular.

15. The cartesian equations of a line are $6x - 2 = 3y + 3 = 2z - 4$. Find the direction ratios of the line.

16. Find the integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{\sqrt{1-x^2}} = x$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. In a family there are four children. All of them have to work in fields to earn their livelihood at the age of 15.

Based on the above information, answer the following questions :

(i) Probability that all children working in fields are boys if it is given that elder child working in fields is a boy, is

(a) 3/8 (b) 1/8
(c) 5/8 (d) none of these

(ii) Probability that all children working in fields are grass, if first two children working in fields are girls, is

(a) 1/4 (b) 3/4 (c) 1/2 (d) none of these

(iii) Find the probability that two middle child working in fields are boys if it is given that first child working in fields is a girl.

(a) 0 (b) 3/4 (c) 1/4 (d) none of these

(iv) Find the probability that all children working in fields are girls if it is given that at least one of the children working in fields is a girl.

(a) 0 (b) 1/15 (c) 2/15 (d) 4/15

(v) Find the probability that all children working in fields are boys if it is given that at least three of the children working in fields are boys.

(a) 1/5 (b) 2/5 (c) 3/5 (d) 4/5

18. In a street two lamp posts are 300 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source).

The combined light intensity is the sum of the two light intensities coming from both lamp posts.
Based on the above information, answer the following.

(i) If you are in between the lamp posts, at distance x feet from the stronger light, then the formula for the combined light intensity coming from both lamp posts as function of x, is

(a) $\frac{1000}{x^2}+\frac{125}{x^2}$ (b) $\frac{1000}{(300-x^2)}+\frac{125}{x^2}$ (c) $\frac{1000}{x^2}+\frac{125}{(300-x)^2}$ (d) None of these

(ii) The maximum value of x can not be

(a) 100 (b) 200 (c) 300 (d) None of these

(iii) The minimum value of x can not be

(a) 0 (b) 100 (c) 200 (d) None of these

(iv) If $I(x)$ denote the combined light intensity, then $I(x)$ will be minimum when $x =$

(a) 100 (b) 200 (c) 300 (d) 150

(v) The darkest spot between the two lights is

(a) at a distance of 100 feet from the weaker lamp post.
(b) at distance of 100 feet from the stronger lamp post.
(c) at a distance of 200 feet from the weaker lamp post.
(d) None of these

PART – B

Section - III

19. Evaluate $\int_2^4 \frac{x}{x^2+1}dx$ by using substitution method.

20. If $y^2 = ax^2 + bx + c$, then find the value of $\frac{d}{dx}\left(y^3 y_2\right)$.

OR

Differentiate $e^x \log(\sin 2x)$ w.r.t. x.

21. A bag contains 6 red, 5 blue and 7 white balls. If three balls are drawn one by one (without replacement), then what is the probability that all three balls are blue?

22. Determine the area enclosed between the curve $y = \cos^2 x,\ 0 \le x \le \frac{\pi}{2}$ and the axes.

23. Find a unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$.

OR

Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{i}+\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i}-\hat{j}+4\hat{k}$.

24. Solve the differential equation $\frac{dy}{dx}+1 = e^{x+y}$.

25. An unbiased dice is thrown twice. Let the event A be 'odd number on the first throw' and B be the event 'odd number on the second throw'. Check the independence of the events A and B.

26. Evaluate : $\sin\left[2\cos^{-1}\left(\frac{-3}{5}\right)\right]$

27. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then show that $A^2 + I \neq A(A^2 - I)$.

28. Find the values of x if $f(x) = 6(x^2 - 5x - 24)$ is an increasing function.

OR

A rod 108 metres long is bent to form a rectangle. Find its dimensions, if its area is maximum.

Section - IV

29. If the tangent at $P(1, 1)$ on $y^2 = x(2 - x)^2$ meets the curve again at Q, then find the point Q.

30. Show that the function $f : R \to R$ given by $f(x) = x^3 + x$ is bijective.

31. If $\frac{xdy}{dx} + 2y = \ln x$, then find the value of $e^2\, y(e) - y(1)$.

OR

Solve the initial value problem $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0, y(1) = 2$.

32. Find the value of k, for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$ is continuous at $x = 0$.

33. Find the area bounded by the circle $x^2 + y^2 = 8x$ and the line $x = 2$.

34. Evaluate : $\int \frac{x^2 + 9}{x^4 - 2x^2 + 81} dx$

OR

Evaluate : $\int_0^{\pi/2} x^2 \sin^2 x\, dx$

35. Find the second order derivative of $a \sin^3 t$ with respect to $a \cos^3 t$ at $t = \pi/4$.

Section - V

36. Solve the following problem graphically :

Maximize $Z = 22x + 18y$

Subject to constraints :

$x + y \le 20$

$36x + 24y \le 576$

$x, y \ge 0$

OR

Find the maximum value of $z = 3x + 5y$ subject to $x + 4y \le 24$, $3x + y \le 21$, $x + y \le 9$, $x \ge 0$, $y \ge 0$.

37. Find the equation of the plane containing the lines $\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = \hat{i} + \hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k})$. Find the distance of this plane from origin and also from the point (1, 1, 1).

OR

Find the length of the perpendicular drawn from the point (2, 4, −1) to the line $\vec{r} = \hat{i} + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$.

38. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, then find A^{-1} and hence solve the system of linear equations $x + 2y + z = 4$, $-x + y + z = 0$, $x - 3y + z = 2$.

OR

Solve the following system of equations :

$3x - y + z = 5$

$2x - 2y + 3z = 7$

$x + y - z = -1$