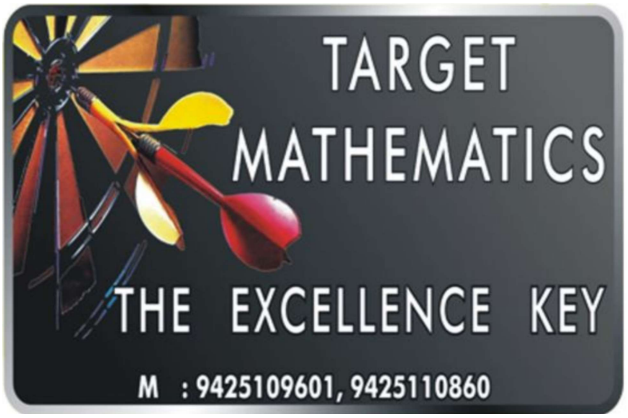
MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*



Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

Target Mathematics
by
Dr. Agyat Gupta

PART - A

Section - I

1. Simplify : $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

OR

If $A^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, then find the value of $A^T - B^T$.

2. Write the projection of the vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$.

3. Evaluate : $\int[\sin(\log x)+\cos(\log x)]dx$

OR

Evaluate : $\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx$

4. If $A = \{0, 1\}$ and N be the set of all natural numbers. Then, show that the mapping $f: N\to A$ defined by $f(2n-1)=0, f(2n)=1\ \forall n\in N$, is onto.

5. Write the principal value branch of $\text{cosec}^{-1}x$.

OR

Find the principal values of $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$.

6. The position vectors of points A and B are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$ respectively, then find the direction cosine of $\overline{AB}$ along Y-axis.

7. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles 30° and 120° respectively, then find $|\vec{a}+\vec{b}|$.

OR

Write a unit vector in the direction of the vector $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$.

8. Find the co-factor of each element of the first column of matrix $A=\begin{bmatrix}2 & 5 & -1\\ -3 & 0 & 1\\ 1 & 1 & -1\end{bmatrix}$.

9. A line makes angle $\frac{\pi}{3}$ with X-axis, $\frac{2\pi}{3}$ with Y-axis and $\frac{\pi}{4}$ with Z-axis. Find the direction cosines of the line.

OR

Find the direction cosines of the line joining the points (4, 3, −5) and (−2, 1, −8).

10. Find the equation of the plane passing through (2, 3, −1) and is perpendicular to the vector $3\hat{i}-4\hat{j}+7\hat{k}$.

11. Find the value of $\tan\left(\sin^{-1}\frac{3}{5}\right)$.

12. Evaluate : $\int_1^2 \frac{x^3-1}{x^2}dx$

13. If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, then find the value of λ.

14. If $A=\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$, then find adj A.

15. Find the vector equation of the plane whose Cartesian equation is $5x - 7y + 2z = 3$.

16. Find the order and degree of the differential equation $y'''+y^2+e^{y'}=0$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Suppose your friend is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time.

Based on the above information, answer the following questions :

(i) If an ordinary year is considered, the probability that it rains on wedding day is

(a) $\frac{1}{365}$ (b) $\frac{1}{73}$ (c) $\frac{1}{72}$ (d) $\frac{6}{365}$

(ii) The probability that it does not rain on wedding day is

(a) $\frac{1}{365}$ (b) $\frac{5}{365}$ (c) $\frac{360}{365}$ (d) none of these

(iii) The probability that the weatherman predicts correctly is

(a) $\frac{5}{10}$ (b) $\frac{7}{10}$ (c) $\frac{9}{10}$ (d) $\frac{1}{10}$

(iv) The probability that it will rain on the wedding day, if weatherman predict rain for tomorrow, is

(a) 0.111 (b) 0.222 (c) 0.333 (d) 0.444

(v) The probability that it will not rain on the wedding day, if weatherman predict rain for tomorrow, is

(a) 0.889 (b) 0.778 (c) 0.667 (d) 0.556

18. A real estate company is going to build a new apartment complex. The land they have purchased can hold at most 5000 apartments. Also, if they make x apartments, then the maintenance costs for the building, landscaping etc., would be as follows:

Fixed cost = ₹ 40,00,000

Variable cost = ₹$(140x - 0.04x^2)$

Based on the above information, answer the following questions :

(i) The maintenance cost as a function of x will be

(a) $14x - 0.04x^2$ (b) 4000000

(c) $4000000 + 140x - 0.04x^2$ (d) None of these

(ii) If $C(x)$ denote the maintenance cost function, then maximum value of $C(x)$ occur at x =

(a) 0 (b) 1750 (c) 5000 (d) 2000

(iii) The maximum value of $C(x)$ would be

(a) ₹ 5225000 (b) ₹ 4122500 (c) ₹ 5000000 (d) ₹ 4000000

(iv) The number of apartments, that the complex should have in order to minimize the maintenance costs, is

(a) 5000 (b) 4000 (c) 1750 (d) 3500

(v) If the minimum maintenance cost is attain, then the maintenance cost for each apartment would be

(a) ₹ 740 (b) ₹ 540 (c) ₹ 640 (d) ₹ 840

PART - B

Section - III

19. Find the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing.

20. If $|\vec{a}| = 2, |\vec{b}| = 7$ and $(\vec{a} \times \vec{b}) = 3\hat{i} + 2\hat{j} + 6\hat{k}$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find a unit vector in the direction of the resultant of vectors $\hat{i} + 2\hat{j} + 3\hat{k}$, $-\hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j}$.

21. If $A = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix}$, then find $A + A^{-1}$.

22. Mother, father and son line up at random for a family picture. Find $P(A/B)$, if A and B are defined as follows :

A = Son on one end, B = Father in the middle

23. Evaluate : $\int \frac{dx}{3\sin^2 x + 4}$

OR

Evaluate $\int_{\pi/3}^{\pi/4} (\tan x + \cot x)^2 \, dx$ by fundamental theorem of integral calculus.

24. Let $f: R \to R$ be a function defined by $f(x)=\frac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$, then show that $f(x)$ is many one into function.

25. Solve the differential equation $\frac{dy}{dx}=e^{x+y}+e^{y}x^{3}$.

26. Find the area bounded by $y^2 = x$, $y = 0$, $x = 1$ and $x = 3$.

27. In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at random. Find the probability that she fails in physics if she has failed in mathematics.

28. If $f(x)=\begin{cases}\left(\frac{\frac{1}{e^{x}}-1}{\frac{1}{e^{x}}+1}\right), & \text{when } x \neq 0 \\ 0, & \text{when } x=0\end{cases}$, then show that $f(x)$ is discontinuous at $x = 0$.

OR

If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

Section-IV

29. Given the sum of the perimeter of a square and a circle. Show that sum of their areas is least when the side of the square is equal to the diameter of the circle.

30. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the X-axis.

31. If $x^p y^q = (x + y)^{p+q}$, then prove that $\frac{dy}{dx}=\frac{y}{x}$.

OR

If $y=\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$, then show that $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=2$.

32. Evaluate : $\int_0^{\frac{3}{2}} |x\cos \pi x|\,dx$

33. Show that relation 'is congruent to', on the set of all triangles in a plane is an equivalence relation.

34. Solve the differential equation : $(x^2 + 1)\, y' - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 0$.

OR

Solve the differential equation $x(x^2-1)\frac{dy}{dx}=1$, given that when $x = 2$, $y = 0$.

35. Find the values of a, b respectively if $f(x)=\begin{cases}x^2+3x+a, & x \leq 1 \\ bx+2, & x>1\end{cases}$ is differentiable at every x.

Section-V

36. Find the matrix P satisfying the matrix equation $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix} P \begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

OR

Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of ₹ x, ₹ y and ₹ z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of ₹ 29,000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of ₹ 30,500. If the three prizes per person together cost ₹ 9,500; then

(i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication.

(ii) Solve these equations using matrices.

37. Find the distance of the point (–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

OR

Find the equation of the plane passing through three given points $-2\hat{i}+6\hat{j}-6\hat{k}, -3\hat{i}+10\hat{j}-9\hat{k}, -5\hat{i}-6\hat{k}$.

38. Solve the following linear programming problem (LPP) graphically.

Maximize $Z = \frac{35}{2}x + 7y$

Subject to constraints :

$x + 3y \leq 12$;

$3x + y \leq 12$;

$x, y \geq 0$

OR

Solve the following linear programming problem (LPP) graphically.

Maximize $Z = 500\,x + 150\,y$

Subject to constraints :

$2500\,x + 500\,y \leq 50000$

$x + y \leq 60$;

$x, y \geq 0$

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