TARGET MATHEMATICS

THE EXCELLENCE KEY

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Sample Paper

AG-TMC-TS-TERM-1- 004

Time : 90 Minutes **Max Marks : 40**

General Instructions

1. *This question paper contains three sections – A, B and C. Each part is compulsory.*
2. *Section-A has **20** MCQs, attempt any **16** out of 20.*
3. *Section-B has **20** MCQs, attempt any **16** out of 20.*
4. *Section-C has **10** MCQs, attempt any **8** out of 10.*
5. *All questions carry equal marks.*
6. *There is no negative marking.*

SECTION-A

*In this section, attempt **any 16** questions out of questions 1-20. Each question is of 1 mark weightage.*

1. If R is a relation in a set A such that $(a, a) \in R$ for every $a \in A$, then the relation R is called

(a) symmetric (b) reflexive (c) transitive (d) symmetric or transitive

2. If matrix $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = \begin{cases} 1 & \text{if} \quad i \neq j \\ 0 & \text{if} \quad i = j \end{cases}$, then A^2 is equal to

(a) I (b) A (c) O (d) None of these

3. The value of $\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is

(a) $a^3 + b^3 + c^3$ (b) $3bc$

(c) $a^3 + b^3 + c^3 - 3abc$ (d) None of these

4. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

(a) two points of local maximum (b) two points of local minimum

(c) one maxima and one minima (d) no maxima or minima

5. If $|x - 1| > 5$, then

(a) $x \in (-4, 6)$ (b) $x \in [-4, 6]$

(c) $x \in (-\infty, -4) \cup (6, \infty)$ (d) $x \in (-\infty, -4) \cup (6, \infty)$

6. The maximum value of $\sin x . \cos x$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

7. Let A, B and C are three matrices of same order. Now, consider the following statements

I. If A = B, then AC = BC

II. If AC = BC, then A = B

Choose the correct option

(a) Only I is true (b) Only II is true

(c) Both I and II are true (d) Neither I nor II is true

8. If the area of a triangle with vertices (– 3, 0), (3, 0) and (0, k) is 9 sq. units. Then, the value of k will be

(a) 9 (b) 3 (c) – 9 (d) 6

9. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Then, R is

(a) reflexive (b) symmetric

(c) transitive (d) an equivalence relation

10. The value of $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$ is

(a) $9x^2(x+y)$ (b) $9y^2(x+y)$ (c) $3y^2(x+y)$ (d) $7x^2(x+y)$

11. The function $f(x) = x^x$ has a stationary point at

(a) $x = e$ (b) $x = \frac{1}{e}$ (c) $x = 1$ (d) $x = \sqrt{e}$

12. If $|x + 2| \le 9$, then

(a) $x \in (-7, 11)$ (b) $x \in [-11, 7]$

(c) $x \in (-\infty, -7) \cup (11, \infty)$ (d) $x \in (-\infty, -7) \cup [11, \infty)$

13. The maximum value of $\left(\frac{1}{x}\right)^x$ is

(a) e (b) e^e (c) $e^{\frac{1}{e}}$ (d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

14. If A and B are 2 × 2 matrices, then which of the following is true?

(a) $(A+B)^2 = A^2 + B^2 + 2AB$ (b) $(A-B)^2 = A^2 + B^2 - 2AB$

(c) $(A-B)(A+B) = A^2 + AB - BA - B^2$ (d) $(A+B)(A-B) = A^2 - B^2$

15. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value of x is

(a) 3 (b) ± 3 (c) ± 6 (d) 6

16. The function $f(x) = \tan x - x$

(a) always increases (b) always decreases

(c) never increases (d) sometimes increases and sometimes decreases

17. For the set A = {1, 2, 3}, define a relation R in the set A as follows R = {(1, 1), (2, 2), (3, 3), (1, 3)}. Then, the ordered pair to be added to R to make it the smallest equivalence relation is

(a) (1,3) (b) (3, 1)
(c) (2,1) (d) (1,2)

18. The equation of the normal to the curve $y^4 = ax^3$ at (a, a) is

(a) $x + 2y = 3a$ (b) $3x - 4y + a = 0$
(c) $4x + 3y = 7a$ (d) $4x - 3y = 0$

19. If A is an invertible matrix of order 2, then det. (A^{-1}) is equal to :

(a) det. (A) (b) $\frac{1}{\text{det.}(A)}$ (c) 1 (d) 0

20. The equation of the tangent to curve $y = be^{-x/a}$ at the point where it crosses y-axis is

(a) $ax + by = 1$ (b) $ax - by = 1$
(c) $\frac{x}{a} - \frac{y}{b} = 1$ (d) $\frac{x}{a} + \frac{y}{b} = 1$

SECTION-B

In this section, attempt ***any 16*** *questions out of the questions 21-40. Each question is of 1 mark weightage.*

21. If A and B are two matrices such that A + B and AB are both defined, then

(a) A and B are two matrices not necessarily of same order. (b) A and B are square matrices of same order.
(c) Number of columns of A = Number of rows of B. (d) None of these.

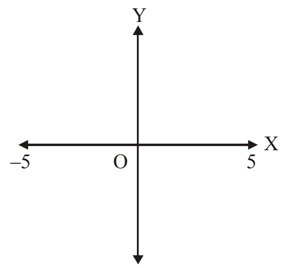
22. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then

(a) $f(a) = 0$ (b) $f(b) = 0$
(c) $f(0) = 0$ (d) $f(1) = 0$

23. Let A = {1, 2, 3} and R = {(1, 2), (2, 3)} be a relation in A. Then, the minimum number of ordered pairs may be added, so that R becomes an equivalence relation, is

(a) 7 (b) 5 (c) 1 (d) 4

24. The inequality representing the following graphs is



(a) $|x| < 5$ (b) $|x| \le 5$ (c) $|x| > 5$ (d) $|x| \ge 5$

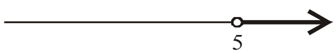
25. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ is (θ is real number)

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\frac{2\sqrt{3}}{4}$

26. If $A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$, then A^{-1} exists, if

(a) $1 = 2$ (b) $1 \neq 2$ (c) $1 \neq -2$ (d) None of these

27. Solution of a linear inequality in variable x is represented on number line is



(a) $x \in (-\infty, 5)$ (b) $x \in (-\infty, 5]$ (c) $x \in [5, \infty)$ (d) $x \in (5, \infty)$

28. If there are two values of a which makes determinant,

$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then the sum of these numbers is

(a) 4 (b) 5 (c) -4 (d) 9

29. The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is :

(a) $\frac{22}{7}$ (b) $\frac{6}{7}$ (c) $\frac{-6}{7}$ (d) -6

30. If $A = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 \\ 7 & 2 \end{pmatrix}$ then which statement is true ?

(a) $AA^T = I$ (b) $BB^T = I$ (c) $AB \neq BA$ (d) $(AB)^T = I$

31. Let $A = \{1, 2, 3\}$. Then find the number of relations containing $(1, 2)$ and $(1, 3)$, which are reflexive and symmetric but not transitive, is

(a) 1 (b) 2 (c) 3 (d) 4

32. If A and B are invertible matrices, then which of the following is not correct?

(a) $\text{adj}\, A = |A| . A^{-1}$ (b) $\det (A)^{-1} = [\det (A)]^{-1}$ (c) $(AB)^{-1} = B^{-1} A^{-1}$ (d) $(A + B)^{-1} = B^{-1} + A^{-1}$

33. The normal to the curve $x = a(1 + \cos\theta)$, $y = a \sin\theta$ at 'θ' always passes through the fixed point

(a) (a, a) (b) $(0, a)$ (c) $(0, 0)$ (d) $(a, 0)$

34. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, if the value of α is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$

35. If the constraints in a linear programming problem are changed

(a) The problem is to be re-evaluated (b) Solution is not defined

(c) The objective function has to be modified (d) The change in constraints is ignored

36. The slope of the normal to the curve $y^3 - xy - 8 = 0$ at the point (0, 2) is equal to

(a) -3 (b) -6 (c) 3 (d) 6

37. If there are two values of a which makes determinant,

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$$, then the sum of these numbers is

(a) 4 (b) 5 (c) -4 (d) 9

38. If the number of available constraints is 3 and the number of parameters to be optimized is 4, then

(a) The objective function can be optimized (b) The constraint are short in number

(c) The solution is problem oriented (d) None of these

39. The curve $y - e^{xy} + x = 0$ has a vertical tangent at

(a) (1, 1) (b) (0, 1) (c) (1, 0) (d) no point

40. If A is any square matrix, then which of the following is skew-symmetric?

(a) $A + A^T$ (b) $A - A^T$ (c) AA^T (d) A^TA

SECTION-C

*In this section, attempt **any 8** questions. Each question is of 1 mark weightage. Questions 46-50 are based on a case-study.*

41. Let $f(x) = \begin{cases} 3x - 4, & 0 \le x \le 2 \\ 2x + \ell, & 2 < x \le 9 \end{cases}$

If f is continuous at x = 2, then what is the value of ℓ ?

(a) 0 (b) 2 (c) -2 (d) -1

42. If $f(x) = \dfrac{\sqrt{4+x} - 2}{x}$, $x \ne 0$ be continuous at x = 0, then f(0) =

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 2 (d) $\frac{3}{2}$

43. If the function $f(x) = \begin{cases} 1 & , \quad x \le 2 \\ ax + b & , \quad 2 < x < 4 \\ 7 & , \quad x \ge 4 \end{cases}$

is continuous at x = 2 and 4, then the values of a and b are.

(a) a = 3, b = –5 (b) a = –5, b = 3 (c) a = –3, b = 5 (d) a = 5, b = –3

44. The number of solutions of the equation $3\tan x + x^3 = 2$ in $\left(0, \frac{\pi}{4}\right)$ is

(a) 1 (b) 2 (c) 3 (d) infinite

45. The difference between greatest and least value of $f(x) = 2\sin x + \sin 2x, \quad x \in \left[0, \frac{3\pi}{2}\right]$ is –

(a) $\frac{3\sqrt{3}}{2}$ (b) $\frac{3\sqrt{3}}{2} - 2$

(c) $\frac{3\sqrt{3}}{2} + 2$ (d) None of these

Case Study

A teacher prepared a performance grade criteria for +2 students on the basis of the numbers of x hours devoted by the students.

$$f(x) = \begin{cases} 1 \quad , \text{ if } \quad x \le 3 \\ ax + b, \text{ if } \; 3 < x < 5 \\ 7 \quad , \text{ if } \quad x \ge 5 \end{cases} \Rightarrow \begin{cases} \text{Grade 1, unsatisfactory } x \le 3 \\ \text{Grade } (ax + b)\text{, satisfactory } x = 4 \\ \text{Grade 7, Average } x \ge 5 \end{cases}$$

Based on the above information answer the following :

46. If f(x) is continuous at x = 3 then relation between a and b is

(a) $5a + b = 7$ (b) $3a + b = 1$ (c) $5a + b = 1$ (d) $3a + b = 7$

47. If f(x) is continuous at x = 5 then relation between a and b is

(a) $5a + b = 7$ (b) $5a + b = 1$ (c) $3a + b = 7$ (d) $3a + b = 1$

48. The value of a and b are

(a) 2, 5 (b) 3, 8 (c) 3, –8 (d) 8, 3

49. If satisfactory level is x = 4 then grade is

(a) 4 (b) 1 (c) 7 (d) 0

50. If satisfactory level is x = 10 then grade is

(a) 4 (b) 1 (c) 0 (d) 7

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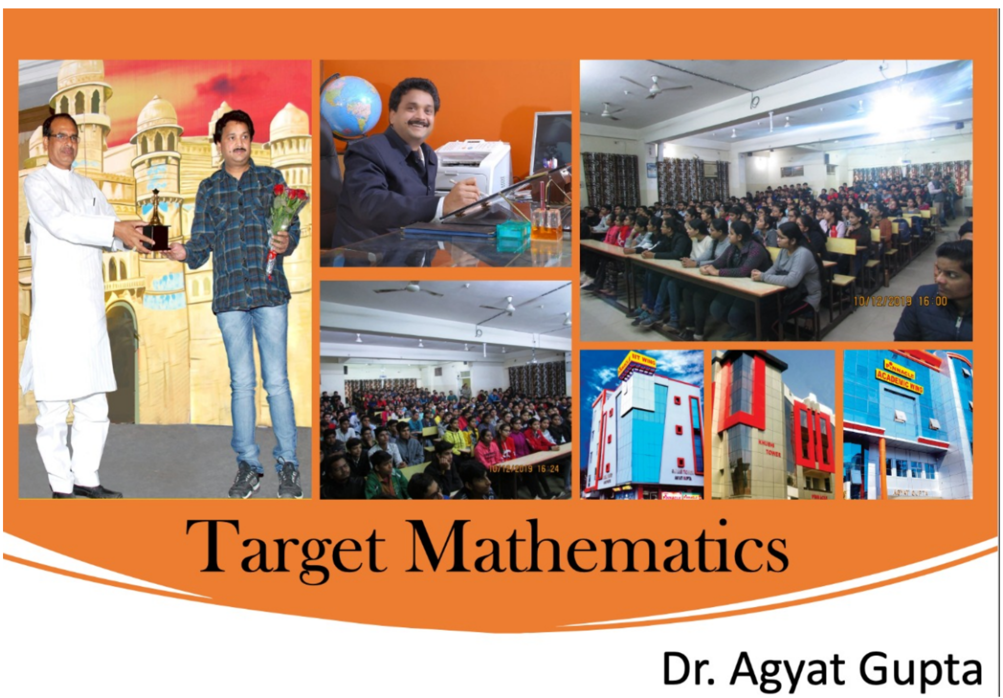

AG-TMC-TS-TERM-1-004- ANSWER SHEET

Sample Paper 4

ANSWER KEYS																			
1	(b)	6	(b)	11	(b)	16	(a)	21	(b)	26	(d)	31	(a)	36	(b)	41	(c)	46	(b)
2	(a)	7	(a)	12	(b)	17	(b)	22	(c)	27	(d)	32	(d)	37	(c)	42	(b)	47	(a)
3	(d)	8	(b)	13	(c)	18	(c)	23	(a)	28	(c)	33	(d)	38	(b)	43	(a)	48	(c)
4	(c)	9	(d)	14	(c)	19	(b)	24	(a)	29	(b)	34	(b)	39	(c)	44	(a)	49	(a)
5	(c)	10	(b)	15	(c)	20	(d)	25	(a)	30	(d)	35	(a)	40	(b)	45	(c)	50	(d)

SOLUTIONS

1. **(b)** A relation R in a set A is called reflexive, if $(a, a) \in R$ for every $a \in A$.

2. **(a)** $a_{11} = 0, a_{12} = 1, a_{21} = 1, a_{22} = 0$

$$\therefore A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

3. **(d)** **4.** **(c)**

5. **(c)** Since, $|x-1| > 5$ So, $(x-1) < -5$ or $(x-1) > 5$

$[\,|x| > a \Rightarrow x < -a \text{ or } x > a\,]$

Therefore, $x < -4$ or $x > 6$

Hence, $x \in (-\infty, -4) \cup (6, \infty)$

6. **(b)**

7. **(a)** For three matrices A,B and C of the same order, if A = B, then AC = BC but the converse is not true.

8. **(b)**

9. **(d)** Clearly, (x, y) R (x, y) $\forall$ (x, y) $\in$ A, since xy = yx. This shows that R is reflexive. Further (x, y) R (u, v)

$\Rightarrow$ xv = yu

$\Rightarrow$ uy = vx and hence (u, v) R (x, y). This shows that R is symmetric. Similarly, (x, y) R (u, v) and (u, v) R (a, b).

$\Rightarrow$ xv = yu and ub = va $\Rightarrow xv\frac{a}{u} = yu\frac{a}{u} \Rightarrow xv\frac{b}{v} = yu\frac{a}{u}$

$\Rightarrow$ xb = ya and hence (x, y) R (a, b). Therefore, R is transitive.

Thus, R is an equivalence relation.

10. **(b)**

11. **(b)** Since, $f(x) = x^x$

Suppose $y = x^x \quad \therefore \log y = x \log x$

After differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x \quad \text{So, } \frac{dy}{dx} = (1 + \log x)x^x$$

Now, $\frac{dy}{dx} = 0 \Rightarrow (1 + \log x)\,.\,x^x = 0$

$$\Rightarrow \log x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$$

Hence, $f(x)$ has a stationary point at $x = \frac{1}{e}$

12. **(b)** Given, $|x + 2| \le 9$

$\Rightarrow -9 \le x + 2 \le 9$

$\Rightarrow -11 \le x \le 7$

13. **(c)**

14. **(c)** Given that, A and B are 2 × 2 matrices.

$\therefore (A - B) \times (A + B) = A \times A + A \times B - B \times A - B \times B$

$= A^2 - B^2 + AB - BA$

$\Rightarrow (A - B)(A + B) = A^2 + AB - BA + B^2$

15. **(c)** **16.** **(a)**

17. **(b)** The given relation is R = {(1, 1), (2, 2),(3, 3), (1, 3)} on the set A = {1, 2, 3}.

Clearly, R is reflexive and transitive.

To make R symmetric, we need (3, 1) as (1, 3) $\in$ R.

$\therefore$ If (3, 1) $\in$ R, then R will be an equivalence relation. Hence, (3, 1) is the single ordered pair which needs to be added to R to make it the smallest equivalence relation.

18. **(c)**

19. **(b)** $|A| \neq 0$

$\Rightarrow A^{-1}$ exists $\Rightarrow AA^{-1} = I \Rightarrow |AA^{-1}| = |I| = 1$

$\Rightarrow |A||A^{-1}| = 1 \quad |A^{-1}| = \frac{1}{|A|}$

Hence option (b) is correct.

20. **(d)** Curve is $y = be^{-x/a}$

Since the curve crosses y-axis (*i.e.*, $x = 0$)

$\therefore y = b$

Now $\frac{dy}{dx} = \frac{-b}{a}e^{-x/a}$. At point $(0, b)$, $\left(\frac{dy}{dx}\right)_{(0,b)} = \frac{-b}{a}$

$\therefore$ equation of tangent is, $y - b = \frac{-b}{a}(x - 0)$

$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1.$

21. **(b)** A + B is defined $\Rightarrow$ A and B are of same order.

Also AB is defined $\Rightarrow$

Number of columns in A = Number of rows in B

Obviously, both simultaneously mean that the matrices A and B are square matrices of same order.

22. **(c)**

23. **(a)** The given relation is $R = \{(1, 2), (2, 3)\}$ in the set $A = \{1, 2, 3\}$.

Now, R is reflexive, if $(1, 1), (2, 2), (3, 3) \in R$.

R is symmetric, if $(2, 1), (3, 2) \in R$.

R is transitive, if $(1, 3)$ and $(3, 1) \in R$.

Thus, the minimum number of ordered pairs which are to be added, so that R becomes an equivalence relation, is 7.

24. **(a)** The graph represents $x > -5$ and $x < 5$. So, $|x| < 5$.

25. **(a)**

26. **(d)** Since, $A = \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix}$

After expanding along R_1, we get

$|A| = 2(6-5) - \lambda(-5) - 3(-2) = 5\lambda + 8$

As, A^{-1} exists, so $|A| \neq 0 \therefore 5\lambda + 8 \neq 0$

So, $\lambda \neq \frac{-8}{5}$

27. **(d)** **28.** **(c)** **29.** **(b)**

30. **(d)** Here $AA^T = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}\begin{pmatrix} 2 & -7 \\ -1 & 4 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$(BB^T)_{11} = (4)^2 + (1)^2 \neq 1$

$(AB)_{11} = 8 - 7 = 1, (BA)_{11} = 8 - 7 = 1$

$\therefore$ AB $\neq$ BA may be not true.

Now, $AB = \begin{pmatrix} 2 & -1 \\ -7 & 4 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 7 & 2 \end{pmatrix}$

$= \begin{pmatrix} 8-7 & 2-2 \\ -28+28 & -7+8 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; (AB)^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

31. **(a)** Let R be a relation containing (1, 2) and (1, 3) R is reflexive, if $(1, 1), (2, 2), (3, 3) \in R$.

Relation R is symmetric, if $(2, 1) \in R$ but $(3, 1) \notin R$.

But relation R is not transitive as $(3,1), (1,2) \in R$ but $(3,2) \notin R$.

Now, if we add the pair (3, 2) and (2, 3) to relation R, then relation R will become transitive.

Hence, the total number of desired relations is one.

32. **(d)** It is given that A and B are invertible matrices

So, $A^{-1} = \frac{\text{adj } A}{|A|} \therefore \text{adj } A = |A| . A^{-1}$

Now, $\det(A)^{-1} = [\det(A)]^{-1}$ and $(AB)^{-1} = B^{-1}A^{-1}$

and $(A + B)^{-1} \neq B^{-1} + A^{-1}$

33. **(d)** $\frac{dx}{d\theta} = -a\sin\theta$ and $\frac{dy}{d\theta} = a\cos\theta$

$\therefore \frac{dy}{dx} = -\cot\theta.$

$\therefore$ the slope of the normal at $\theta = \tan\theta$

$\therefore$ the equation of the normal at θ is

$y - a\sin\theta = \tan\theta(x - a - a\cos\theta)$

$\Rightarrow y\cos\theta - a\sin\theta\cos\theta = x\sin\theta - a\sin\theta - a\sin\theta\cos\theta$

$\Rightarrow x\sin\theta - y\cos\theta = a\sin\theta$

$\Rightarrow y = (x - a)\tan\theta$

which always passes through (a, 0)

34. **(b)** Now $A + A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad 2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

Thus option (b) is correct.

35. **(a)** **36.** **(b)** **37.** **(c)** **38.** **(b)** **39.** **(c)**

40. **(b)** $(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T)$

Hence, $(A - A^T)$ is skew-symmetric.

41. **(c)** **42.** **(b)** **43.** **(a)** **44.** **(a)** **45.** **(c)**

46. **(b)** **47.** **(a)** **48.** **(c)** **49.** **(a)** **50.** **(d)**

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