


Sample Paper 3

AG-TMC-TS-TERM-1- 003

Time : 90 Minutes **Max Marks : 40**

General Instructions

1. *This question paper contains three sections – A, B and C. Each part is compulsory.*
2. *Section-A has **20** MCQs, attempt any **16** out of 20.*
3. *Section-B has **20** MCQs, attempt any **16** out of 20.*
4. *Section-C has **10** MCQs, attempt any **8** out of 10.*
5. *All questions carry equal marks.*
6. *There is no negative marking.*

SECTION-A

*In this section, attempt **any 16** questions out of questions 1-20. Each question is of 1 mark weightage.*

1. If A is a non-singular matrix of order 3, then $|\text{adj } A| = |A|^n$. Here the value of n is

(a) 2 (b) 4 (c) 6 (d) 8

2. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(a) $-\frac{5\pi}{3}$ (b) $\frac{5\pi}{3}$ (c) $-\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$

3. If x is real number and $|x| < 3$, then

(a) $x \geq 3$ (b) $-3 < x < 3$ (c) $x \leq -3$ (d) $-3 \leq x \leq 3$

4. If $y = e^{x^x}$, then $\cdot \frac{dy}{dx} =$

(a) $y(1+\log_e x)$ (b) $yx^x(1+\log_e x)$ (c) $ye^x(1+\log_e x)$ (d) None of these

5. If x is real, then the minimum value of $x^2 - 8x + 17$ is

(a) – 1 (b) 0 (c) 1 (d) 2

6. If $\sin^{-1} x = y$, then

(a) $0 \leq y \leq \pi$ (b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ (c) $0 < y < \pi$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

7. x and b are real numbers. If $b > 0$ and $|x| > b$, then

(a) $x \in (-b, \infty)$ (b) $x \in (-\infty, b)$ (c) $x \in (-b, b)$ (d) $x \in (-\infty, -b) \cup (b, \infty)$

8. Given : $2x - y - 4z = 2,\ x - 2y - z = -4,\ x + y + \lambda z = 4$, then the value of λ such that the given system of equation has no solution, is

(a) 3 (b) 1 (c) 0 (d) – 3

9. The function $f(x) = \tan x - x$

(a) always increases (b) always decreases
(c) never increases (d) sometimes increases and sometimes decreases

10. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1+\log y}{y \log y}$ (b) $\frac{(1+\log y)^2}{y \log y}$

(c) $\frac{1+\log y}{(\log y)^2}$ (d) $\frac{(1+\log y)^2}{\log y}$

11. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(a) π (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

12. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$?

(a) sin 2x (b) tan x (c) cos x (d) cos 3x

13. L.P.P is a process of finding
(a) Maximum value of objective function
(b) Minimum value of objective function
(c) Optimum value of objective function
(d) None of these

14. If A be a square matrix of order 3×3, then $|kA|$ is equal to
(a) $k|A|$ (b) $k^2\ |A|$ (c) $k^3\ |A|$ (d) $3k\ |A|$

15. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

(a) increasing in $\left(\pi, \frac{3\pi}{2}\right)$ (b) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) decreasing in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) decreasing in $\left[0, \frac{\pi}{2}\right]$

16. Which of the following is the principal value branch of $\text{cosec}^{-1}x$?

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (b) $(0, \pi) - \left[\frac{\pi}{2}\right]$

(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

17. L.P.P. has constraints of
(a) one variables (b) two variables
(c) one or two variables (d) two or more variables

18. Which of the following is correct :
(a) Determinant is a square matrix
(b) Determinant is a number associated to a matrix
(c) Determinant is a number associated to a square matrix
(d) None of these

19. If $y = x(x-3)^2$ decreases for the values of x given by

(a) $1 < x < 3$ (b) $x < 0$ (c) $x > 0$ (d) $0 < x < \frac{3}{2}$

20. If x = f(t) and y = g(t), then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{g''(t)}{f''(t)}$

(b) $\frac{g''(t)f'(t) - g'(t)f''(t)}{(f'(t))^3}$

(c) $\frac{g''(t)f'(t) - g'(t)f''(t)}{(f'(t))^2}$

(d) None of these

SECTION-B

*In this section, attempt **any 16** questions out of the questions 21-40. Each question is of 1 mark weightage.*

21. Corner points of feasible region of inequalities gives

(a) optional solution of L.P.P. (b) objective function

(c) constraints. (d) linear assumption

22. If $f : R \to R$ be definend by $f(x) = 2x + \cos x$, then f

(a) has a minimum at $x = \pi$ (b) has a maximum at $x = 0$

(c) is a decreasing function (d) is an increasing function

23. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

24. If area of triangle is 35 sq. units with vertices (2, – 6), (5, 4) and (k, 4). Then k is

(a) 12 (b) – 2 (c) – 12, – 2 (d) 12, – 2

25. If $y = \frac{f(x)}{\phi(x)}$ and $z = \frac{f'(x)}{\phi'(x)}$, then $\frac{f''}{f} - \frac{\phi''}{\phi} + \frac{2(y-z)}{f\phi}(\phi')^2 =$

(a) $\frac{d^2y}{dx^2}$ (b) $\frac{1}{y}\frac{d^2y}{dx^2}$

(c) $y\frac{d^2y}{dx^2}$ (d) None of these.

26. If $x^2 + y^2 = 1$, then

(a) $yy'' - (2y')^2 + 1 = 0$ (b) $yy'' - (y')^2 + 1 = 0$

(c) $yy'' - (y')^2 - 1 = 0$ (d) $yy'' - 2(y')^2 + 1 = 0$

27. The domain of the function $\cos^{-1}(2x-1)$ is

(a) [0, 1] (b) [–1, 1]

(c) (– 1, 1) (d) [0, π]

28. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $[-1, \infty)$ (b) $[-2, -1]$

(c) $(-\infty, -2]$ (d) $[-1, 1]$

29. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to

(a) π (b) $-\frac{\pi}{2}$ (c) 0 (d) $2\sqrt{3}$

30. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is the cofactors of a_{ij}, then value of Δ is given by

(a) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ (b) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(c) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (d) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

31. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ intersect at an angle of

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

32. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?

(a) $f(x) + g(x)$ (b) $f(x) - g(x)$ (c) $f(x).g(x)$ (d) $\frac{g(x)}{f(x)}$

33. The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is

(a) $\frac{22}{7}$ (b) $\frac{6}{7}$ (c) $\frac{-6}{7}$ (d) -6

34. Which of the following is the principal value branch of $\cos^{-1}x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $(0, \pi)$

(c) $[0, \pi]$ (d) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

35. Let A be a non-singular square matrix of order 3 × 3. Then |Adj A| is equal to :

(a) $|A|$ (b) $|A|^2$ (c) $|A|^3$ (d) $3|A|$

36. The tangent to the curve $y = e^{2x}$ at the point (0, 1) meets X-axis at

(a) (0, 1) (b) $\left(-\frac{1}{2}, 0\right)$

(c) (2, 0) (d) (0, 2)

37. Which of these terms is not used in a linear programming problem?

(a) Slack variables (b) Objective function

(c) Concave region (d) Feasible solution

38. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) [1, 2] (b) [–1, 1]
(c) [0, 1] (d) None of these

39. The points at which the tangent to the curve $y = x^3 - 12x + 18$ are parallel to X-axis are

(a) $(2,-2), (-2,-34)$ (b) $(2, 34), (-2, 0)$
(c) $(0, 34), (-2, 0)$ (d) $(2, 2), (-2, 34)$

40. The optimal value of the objective function is attained at the points

(a) Given by intersection of inequations with axes only (b) Given by intersection of inequations with x- axis only
(c) Given by corner points of the feasible region (d) None of these

SECTION-C

*In this section, attempt **any 8** questions. Each question is of 1 mark weightage. Questions 46-50 are based on a case-study.*

41. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which $A^2 = B$, is

(a) 1 (b) –1 (c) 4 (d) no real values

42. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then AA^T is

(a) Zero matrix (b) I_2 (c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) None of these

43. The order of the single matrix obtained from $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left\{\begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix}\right\}$ is

(a) 2×3 (b) 2×2 (c) 3×2 (d) 3×3

44. If $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$, then A is a

(a) symmetric matrix (b) skew-symmetric matrix
(c) diagonal matrix (d) none of these

45. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then A^{16} is equal to :

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Case Study

The total cost of producing x T.V. sets per day is ₹ $(x^2 - 5x + 4)$ and the price per set at which they may be sold is ₹ $(2x - 5)$. Based on the above information answer the following.

46. The profit function is

(a) $48x + 4$ (b) $x^2 - 4$

(c) $x^2 - 3x + 54$ (d) $-x^2 + 7x - 9$

47. The profit function is

(a) one-one (b) one-many

(c) many-one (d) many-many

48. If 20 units T.V. produced in one day then profit is

(a) ₹ 400 (b) ₹ 35

(c) ₹ 396 (d) None of these

49. The number of T.V. produced in a day such that profit is zero are

(a) 2 units (b) $\pm$ 2 units

(c) 5 units (d) $\pm$ 5 units

50. The minimum number of T.V. produced in a day to make loss are

(a) 2 units (b) 1 unit

(c) 5 units (d) 10 units

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Sample Paper 3

AG-TMC-TS-TERM-1 -ANSWER SHEET -003

ANSWER KEYS																			
1	(a)	**6**	(b)	**11**	(b)	**16**	(d)	**21**	(a)	**26**	(b)	**31**	(c)	**36**	(b)	**41**	(d)	**46**	(b)
2	(c)	**7**	(d)	**12**	(c)	**17**	(d)	**22**	(d)	**27**	(a)	**32**	(d)	**37**	(c)	**42**	(b)	**47**	(a)
3	(b)	**8**	(d)	**13**	(c)	**18**	(c)	**23**	(d)	**28**	(b)	**33**	(b)	**38**	(a)	**43**	(d)	**48**	(c)
4	(b)	**9**	(a)	**14**	(c)	**19**	(a)	**24**	(d)	**29**	(b)	**34**	(c)	**39**	(d)	**44**	(b)	**49**	(a)
5	(c)	**10**	(d)	**15**	(b)	**20**	(b)	**25**	(b)	**30**	(d)	**35**	(b)	**40**	(c)	**45**	(d)	**50**	(b)

SOLUTIONS

1. **(a)** If A is a non singular matrix of order m, then $|\text{adj}(A)| = |A|^{m-1}$. Here m = 3

$\therefore\ |\text{adj}(A)| = |A|^{3-1} = |A|^2\ \therefore\ n = 2$

2. **(c)** Let $\theta = \sin^{-1}\left[\sin\frac{5\pi}{3}\right]$

$\Rightarrow\ \sin\theta = \sin\frac{5\pi}{3} = \sin\left[2\pi - \frac{\pi}{3}\right]$

$\Rightarrow\ \sin\theta = -\sin\frac{\pi}{3} = \sin\left(\frac{-\pi}{3}\right)\ (\because \sin(-\theta) = -\sin\theta)$

Therefore, principal value of $\sin^{-1}\left[\sin\frac{5\pi}{3}\right]$ is $\frac{-\pi}{3}$, as principal value of $\sin^{-1}x$ lies between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$.

3. **(b)** $|x| < 3 \Rightarrow -3 < x < 3$

4. **(b)**

5. **(c)** Since, $f(x) = x^2 - 8x + 17$
After differentiating w.r.t. x, we get
$f'(x) = 2x - 8$ As $f'(x) = 0 \Rightarrow x = 4$
Here, $f''(x) = 2 > 0, \forall x$
Hence, $x = 4$ is point of local minnima
and minimum value of $f(x)$
$f(4) = (4 \times 4) - (8 \times 4) + 17 = 1$

6. **(b)** The range of principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore$ if $\sin^{-1}x = y$ then $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

7. **(d)** We have, $|x| > b, b > 0$
$\Rightarrow\ x < -b$ and $x > b\ \Rightarrow\ x \in (-\infty, -b) \cup (b, \infty)$

8. **(d)**

9. **(a)** Since, $f(x) = \tan x - x$
After differentiating w.r.t. x, we get
$f'(x) = \sec^2 x - 1$ So, $f'(x) > 0, \forall x \in R$
Hence, $f(x)$ is always increases

10. **(d)**

11. **(b)** $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$, $\sec^{-1}(-2) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\because$ Principal value of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore\ \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

12. **(c)** $\because$ f(x) = cos x

$\Rightarrow\ f'(x) = -\sin x < 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$

So, f(x) = cos x is decreasing in $\left(0, \frac{\pi}{2}\right)$

13. **(c)** **14.** **(c)** **15.** **(b)**

16. **(d)** Principal value branch of $\text{cosec}^{-1}x$

$= \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

17. **(d)**

18. **(c)**

19. **(a)** Since, $y = x(x-3)^2$
After differentiating w.r.t. x, we get

$\frac{dy}{dx} = x.2(x-3) + (x-3)^2$

$= 3x^2 - 12x + 9 = 3(x-3)(x-1)$

Hence, $y = x(x-3)^2$ decreases for $1 < x < 3$.

20. **(b)** $\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{g'(t)}{f'(t)}$

Differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{f'(t)g''(t) - g'(t)f''(t)}{(f'(t))^2} \cdot \frac{dt}{dx}$$

$$= \frac{f'(t)g''(t) - g'(t)f''(t)}{(f'(t))^3}$$

21. **(a)**

22. **(d)** Since, $f(x) = 2x + \cos x$ So, $f'(x) = 2 - \sin x$

Therefore, $f'(x) > 0, \forall x \quad [-1 \le \sin x \le 1]$

Hence, $f(x)$ is an increasing function.

23. **(d)** $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

$\therefore \sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right]$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\frac{\pi}{2} = 1$

24. **(d)** **25.** **(b)** **26.** **(b)**

27. **(a)** Here, $-1 \le 2x - 1 \le 1 \qquad [\because -1 \le \cos\theta \le 1]$

$\Rightarrow 0 \le 2x \le 2 \Rightarrow 0 \le x \le 1$. So, $x \in [0, 1]$

Hence, domain of $\cos^{-1}(2x-1)$ is $[0,1]$.

28. **(b)**

29. **(b)** $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\cot^{-1}(-\sqrt{3})$

$\because$ The range of principal value of $\cot^{-1} x$ is $(0, \pi)$

$\therefore \tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} - \left(\frac{5\pi}{6}\right)$

$= \frac{2\pi - 5\pi}{6} = \frac{-3\pi}{6} = -\frac{\pi}{2}$

30. **(d)** **31.** **(c)** **32.** **(d)**

33. **(b)** $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at $(2, -1)$

$\therefore \quad t^2 + 3t - 8 = 2$ ...(i)

$2t^2 - 2t - 5 = -1$ ...(ii)

On solving eqs (i) and (ii) we get $t = 2$

Now $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3}$

$\therefore \left[\frac{dy}{dx}\right]_{t=2} = \frac{6}{7}$

34. **(c)** Principal value branch of $\cos^{-1}x = [0, \pi]$.

35. **(b)** **36.** **(b)**

37. **(c)** In linear programming problem, concave region is not used. Convex region is used in linear programming.

38. **(a)** $\frac{-\pi}{2} \le \sin^{-1}\sqrt{x-1} \le \frac{\pi}{2} \Rightarrow -1 \le \sqrt{x-1} \le 1$

$\Rightarrow 0 \le x - 1 < 1 \Rightarrow 1 \le x \le 2$

$\therefore$ Domain of $f(x)$ is $[1, 2]$

39. **(d)** **40.** **(c)**

41. **(d)** Given that $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

and $A^2 = B$

$\Rightarrow \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 = 1, \alpha + 1 = 5 \quad \Rightarrow \alpha = \pm 1, \alpha = 4$

$\because$ There is no common value

$\therefore$ There is no real value of α for which $A^2 = B$

42. **(b)** We have

$A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \quad \therefore A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

Now $AA^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$= \begin{bmatrix} \cos^2 x + \sin^2 x & \cos x \sin x - \sin x \cos x \\ \sin x \cos x - \cos x \sin x & \sin^2 x + \cos^2 x \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

43. **(d)** When a 3×2 matrix is post multiplied by a 2×3 matrix, the product is a 3×3 matrix.

44. **(b)** $A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = -A$

Since $A^T = -A$, therefore, A is a skew symmetric matrix.

45. **(d)** We have $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is identity matrix

$(A^2)^8 = (-I)^8 = I$. Hence, $A^{16} = I$

46. **(b)** **47.** **(a)** **48.** **(c)** **49.** **(a)** **50.** **(b)**