QUADRATIC EQUATIONS (TERM-2)

Type-4 Questions. (PULKIT JAWAL)

(Nature of roots)

Distinct (Real) roots, Equal roots and No real roots (Imaginary Roots)

Q1. Determine the nature of roots for the following equations:-

(a) $3z^2 - 4\sqrt{3}z + 4 = 0$
(b) $2x^2 + 14x - 240 = 0$
(c) $3x^2 = 2x^2 - \frac{x}{3}$
(d) $(t) + (t)^{-1} = 3$
(e) $-2(2)^{0.5}x + 1 = -2x^2$
(f) $x^2(7)^{0.5} - 6x - 13(7)^{0.5}$
(g) $7 = -20x - 25x^2$
(h) $3m^2 + 2\sqrt{5}m = 5$
(i) $y^2 - 290 = -(y+2)^2$
(j) $x + 2x^2 - 528 = 0$
(k) $2m^2 + 8m + 9 = 0$
(l) $x^2 = ax - b^2$
(m) $25a^2 - 10a + 1 = 0$
(n) $m + \frac{4}{m} = 1$
(o) $\frac{3}{2}x^2 - x + \frac{1}{6} = 0$

Q2. If discriminant is equal to ZERO, then the equation has which type of Roots?

Q3. The nature of roots completely depends on which parameter?

Q4. find the nature of roots of quadratic Equation $\frac{\sqrt{2}}{2}x^2 + \frac{\sqrt{15}}{2}x + \frac{\sqrt{2}}{2} = 0$

Q5. If the equation $(1+m^2)x^2 + (c^2 - a^2) = -2mcx$ has equal roots then show that $c = a\sqrt{(1+m^2)}$.

Q6. Determine the nature of roots of quadratic equation $z^2 - 4z + 5 = 0$

Q7. Determine the value of 'm' for which the given quadratic equation has Real roots $2x^2 + mx + 2 = 0$

Q8. If the discriminant of this ~~equal~~ equation $c + bx + ax^2$ is equal to ZERO then find the value of 'c'.

Q9. If the equation $cx^2 + ax + b = 0$ has equal roots then show that $a = 2(cb)^{0.5}$

Q10. find the value of 'y' for which the equation $3y^2 - 2y + \frac{1}{3} = 0$ has Coincident Roots.

Q11. If α, β are the roots of the equation $x^2 - p(x+1) - c = 0$. then find the value of $(\alpha+1)(\beta+1) = ??$

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Q12. Find the value of 'K' for which the following equations have equal roots.

(a) $Kx(x-2)=-6$

(b) $Kx=-(3+2x^2)$

(c) $4x^3-3kx^2+x=0$

(d) $kx^2+2kx-12x^2-24x+2=0$

(e) $k^2x^2=2(k-1)x-4$

(f) $18x^2+16kx+32=0$

(g) $kx^2+4x^2+kx+x+1=0$

(h) $(k+1)x^2=2(k-1)x-1$

(i) $(2k+1)x^2+2(k+3)x=-k-5$

(j) $kx(x-3)=-9$

(k) $4x^2=(3kx-1)$

Q13. If -4 is the root of equation $x^2=4-ax$ and the equation $x^2+ax+b=0$ has equal roots. Find the ~~values~~ value of $(a/b)=$??

Q14. If the roots of the equation $(a-b)x^2+(b-c)x+(c-a)=0$ are coincident then show that $a=\frac{1}{2}(b+c)$.

Q15. Find the 2 consecutive ~~odd~~ positive integers, sum of whose squares is 1201.

Q16. Find the 2 consecutive odd positive integers, sum of whose squares is 290.

Q17. Find the value of 'm' for which one root of the quadratic equation $mx^2-14x+8=0$ is six times the other.

Q18. If the roots of the equation $x^2(a^2+b^2)-2(bd+ac)x+(c^2+d^2)=0$ are equal then show that $\boxed{\frac{a}{b}=\frac{c}{d}}$

Q19. Calculate the values of 'k' for which the equation $16=-(x^2+kx)$ has equal roots.

Q20. Find the roots of equation $(m+5)^{0.5}+(m+12)^{0.5}=(2m+41)^{0.5}$

Q21. Find the value of 'k' if the roots of the equation $3x^2+5x-k=0$ is differ by two.

Q22. Find the value of 'm' for which the quadratic equations $3y^2-10y+m=0$ has roots and these roots are reciprocal of each other.

Q23. Find 'a' if the sum of roots of quadratic equations $4x^2+8ax+a+9=0$ is equal to their product.

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ANSWERS. (Type-4) (PULKIT JAWAL)

Basic Concept

(1) $D=0$ (Equal & Real roots)

(2) $D>0$ (2 distinct/Real roots)

(3) $D<0$ (Imaginary/Unreal or No real roots)

(a) $\boxed{3z^2-4\sqrt{3}z+4=0}$

$a=3,\ b=-4\sqrt{3},\ c=4$

$D=b^2-4ac=(-4\sqrt{3})^2-4(3)(4)$

$\Rightarrow 48-48=0$

So here $D=0$ hence the above quadratic equation has Equal & Real roots.

(b) $\boxed{2x^2+14x-240=0}$

$\Rightarrow 2(x^2+7x-120)=0$

$\Rightarrow x^2+7x-120=0.$

$D=(49)-4(1)(-120)$

$D=49+480=529$

Here $D>0$. Hence Q.E. has 2 distinct OR Real roots

(c) $\boxed{3x^3=2x^2-\frac{x}{3}}$

$\Rightarrow 9x^3=6x^2-x$

$\Rightarrow 9x^3-6x^2+x=0.$

$\Rightarrow x(9x^2-6x+1)=0$

$\Rightarrow 9x^2-6x+1=0$

$D=0$, Equal/Real roots

(d) $\boxed{t+(t)^{-1}=3}$

$\Rightarrow t+\frac{1}{t}=3$

$\Rightarrow t^2+1=3t$

$\Rightarrow t^2-3t+1=0$

$\Rightarrow D=5$,

Here $D>0$, Hence 2 distinct/Real roots.

(e) $\boxed{-2(2)^{0.5}x+1=-2x^2}$

$\Rightarrow -2\sqrt{2}x+1=-2x^2$

$\Rightarrow 2x^2-2\sqrt{2}x+1=0.$

$\Rightarrow D=0$, Here Equal/Real roots.

(f) $\boxed{x^2(7)^{0.5}-6x-13(7)^{0.5}}$

$\Rightarrow x^2\sqrt{7}-6x-13\sqrt{7}=0$

$\Rightarrow D=400$ $D>0$

2 distinct/Real roots.

(g) $\boxed{7=-20x-25x^2}$

$\Rightarrow 25x^2+20x+7=0$

$\boxed{D=-300}$ $D<0$

Imaginary and unreal roots

(h) $\boxed{3m^2+2\sqrt{5}m=5}$

$\Rightarrow 3m^2+2\sqrt{5}m-5=0$

$\Rightarrow D=80,\ D>0$

2 Distinct roots

(i) $\boxed{y^2-290=-(y+2)^2}$

$\Rightarrow (y+2)^2+y^2-290=0$

$\Rightarrow y^2+4+4y+y^2-290=0$

$\Rightarrow 2y^2+4y-286=0.$

$\Rightarrow y^2+2y-143=0.$

$D=576,\ D>0.$

(j) $\boxed{x+2x^2-528=0}$

$\Rightarrow 2x^2+x-528=0$

$\Rightarrow \boxed{D=4225}$, $D>0$

(k) $\boxed{2m^2+8m+9=0}$

$D=-8$, $D<0$.

Imaginary Roots

(l) $\boxed{x^2=ax-b^2}$

$\Rightarrow \boxed{x^2-ax+b^2=0.}$

$a=1,\ b=-a,\ c=b^2$

$D=(-a)^2-4(1)(b^2)$

$\boxed{D=a^2-4b^2}$

If $a>b$ then Real roots

(m) $\boxed{25a^2-10a+1=0}$

$D=0$, Equal Real roots

(n) $\boxed{m+\frac{4}{m}=1}$

$m^2+4=m$ OR $\boxed{m^2-m+4=0}$

$\boxed{D=-15}$, $D<0$,

Imaginary Roots

(o) $\boxed{\frac{3}{2}x^2-x+\frac{1}{6}=0}$.

$\Rightarrow 6\times\frac{3}{2}x^2-6x+\frac{6}{6}=0$

$\Rightarrow 9x^2-6x+1=0.$

$D=0$, Equal/Real roots

Q2. $D=0$, equation has Real/Equal roots

Q3. The nature of roots Completely depends on Discriminant $(D=b^2-4ac)$

Q4. $\frac{\sqrt{2}}{2}x^2+\frac{\sqrt{15}}{2}x+\frac{\sqrt{2}}{2}=0$

$\Rightarrow \frac{1}{2}(\sqrt{2}x^2+\sqrt{15}x+\sqrt{2})=0$

$\Rightarrow \sqrt{2}x^2+\sqrt{15}x+\sqrt{2}=0$ $(a=\sqrt{2},\ b=\sqrt{15},\ c=\sqrt{2})$

$D=(\sqrt{15})^2-4(\sqrt{2})(\sqrt{2})=15-8=7$

$D=7$ that means $D>0$.

Hence the given Q.E. has 2 distinct real roots

Q5. $(1+m^2)x^2+(c^2-a^2)+2mcx=0.$

$\Rightarrow (1+m^2)x^2+(2mc)x+(c^2-a^2)=0$

$\Rightarrow a=(1+m^2),\ b=2mc,\ c=(c^2-a^2)$

for equal roots $D=0$

$(2mc)^2-4(1+m^2)(c^2-a^2)=0.$

$\Rightarrow 4m^2c^2-4(c^2-a^2+m^2c^2-m^2a^2)=0$

$\Rightarrow \cancel{4m^2c^2}-4c^2+4a^2-\cancel{4m^2c^2}+4m^2a^2=0$

$\Rightarrow 4(-c^2+a^2+m^2a^2)=0$

$\Rightarrow a^2+m^2a^2=c^2$ OR $a^2(1+m^2)=c^2$

OR $c=\sqrt{a^2(1+m^2)}=a\sqrt{(1+m^2)}$

Hence proved.

Q6. $z^2-4z+5=0$

$D=(-4)^2-4(1)(5)=16-20=-4.$

$D=-4$ i.e. $D<0$.

Hence the given Q.E. has imaginary Roots

Q7. $2x^2+mx+2=0.$

For Real Roots $D\geq 0$

$(m)^2-4(2)(2)\geq 0$

$m^2-16\geq 0$ OR $m^2\geq 16$

$m\geq +4$ OR $m\geq -4$.

Q8. $cx^2+ax+b=0.$

$D=(a)^2-4(c)(b)=0.$ OR $a^2=4cb.$

$\boxed{c=\frac{a^2}{4b}}$

Q9. $cx^2+ax+b=0$.

$D=0$ for equal roots

$\Rightarrow a^2-4(c)(b)=0$

OR $a^2=4cb \Rightarrow a=\sqrt{4cb}$

$a=2\sqrt{cb}$ OR $\boxed{a=2(cb)^{0.5}}$

Q10. $\boxed{3y^2-2y+\frac{1}{3}=0}$

$\Rightarrow 9y^2-6y+1=0$.

(Coincident Roots means equal roots)

$D=0$, $(-6)^2-4(9)(1)=0$

$\Rightarrow 36-36=0$.

$\alpha=\frac{-(-6)+\sqrt{D}}{2\times9}$, $\beta=\frac{-(-6)-\sqrt{D}}{2\times9}$

$\alpha=\frac{6+0}{18}=\frac{1}{3}$, $\beta=\frac{6}{18}=\frac{1}{3}$.

$\therefore \boxed{\alpha=\beta=\frac{1}{3}}$ OR $\boxed{y=\frac{1}{3},\frac{1}{3}}$

Q11. $\boxed{x^2-p(x+1)-c=0}$

$x^2-px-p-c=0$.

$x^2-px-(p+c)=0$ $(a=1, b=-p, c=-(p+c))$

$\alpha+\beta=-b/a=\frac{p}{1}=p$.

$\alpha\beta=\frac{-(p+c)}{1}=-(p+c)$

Now $(\alpha+1)(\beta+1)=\alpha(\beta+1)+1(\beta+1)$

$\Rightarrow (\alpha\beta)+(\alpha+\beta)+1=-(p+c)+p+1$

$\Rightarrow -\cancel{p}-c+\cancel{p}+1=\boxed{1-c}$

Q12. for equal roots $D=0$ OR $\boxed{b^2-4ac=0}$

(a) $\boxed{kx(x-2)=-6}$

$kx^2-2kx+6=0$ $(a=k, b=-2k, c=6)$

$\Rightarrow (-2k)^2-4(k)(6)=0$

$\Rightarrow 4k^2-24k=0$. OR $k(4k-24)=0$.

$\boxed{k=0, k=6.}$

(b) $\boxed{kx=-(3+2x^2)}$

$\Rightarrow 3+2x^2+kx=0$

$\Rightarrow 2x^2+kx+3=0$. for equal roots ($D=0$)

$\rightarrow (k)^2-4(2)(3)=0$ OR $k^2=24$.

$k=\sqrt{24}$, $\boxed{k=\pm2\sqrt{6}.}$

(c) $\boxed{4x^3-3kx^2+x=0}$

$\Rightarrow x(4x^2-3kx+1)=0$.

OR $4x^2-3kx+1=0$.

$(-3k)^2-4(4)(1)=0$

$\Rightarrow 9k^2=16$.

$k^2=16/9$ OR $k=\sqrt{\frac{16}{9}}$

$\boxed{k=\pm\frac{4}{3}}$

(Type-4) contd......

(d) $\boxed{kx^2+2kx-12x^2-24x+2=0}$

$kx^2-12x^2+2kx-24x+2=0$

$x^2(k-12)+x(2k-24)+2=0$.

$a=(k-12)$, $b=(2k-24)$, $c=2$

$b^2-4ac=0$

$(2k-24)^2-4(k-12)(2)=0$

$(2(k-12))^2-8(k-12)=0$

$\Rightarrow 4(k-12)^2-8(k-12)=0$.

$\Rightarrow 4(k-12)((k-12)-2)=0$

$\Rightarrow 4(k-12)=0$ and $((k-12)-2)=0$.

$\Rightarrow 4(k-12)=0$

$\Rightarrow k-12=0$ OR $k=12$

$(k-12-2)=0$.

$k=14$

(e) $\boxed{k^2x^2=2(k-1)x-4}$

$\Rightarrow k^2x^2=2kx-2x-4$.

$\Rightarrow k^2x^2-2(k-1)x+4=0$.

$a=k^2$, $b=-2(k-1)$, $c=4$

$b^2-4ac=0$ $(-2(k-1))^2-4(k^2)(4)=0$

$\Rightarrow 4(k-1)^2-16k^2=0$.

$\Rightarrow 3k^2+2k-1=0$.

$(3k-1)(k+1)=0$

$k=\frac{1}{3},-1$

(f) $\boxed{18x^2+16kx+32=0}$

$9x^2+8kx+16=0$.

$(8k)^2-4(9)(16)=0$ $\therefore$

$64k^2=4\times9\times16$.

$k^2=9$.

$\therefore k=\pm3$

(g) $\boxed{kx^2+4x^2+kx+x+1=0.}$

$x^2(k+4)+x(k+1)+1=0$

$\Rightarrow (k+1)^2-4(k+4)(1)=0$.

$\Rightarrow (k^2+1+2k)-4(k+4)=0$

$\Rightarrow k^2+1+2k-4k-16=0$

$\Rightarrow k^2-2k-15=0$.

$\Rightarrow (k+5)(k-3)=0$

$\Rightarrow \boxed{k=-5,3}$

(h) $\boxed{(k+1)x^2-2(k-1)x+1=0}$

$\rightarrow (-2(k-1))^2-4(k+1)(1)=0$.

$\rightarrow 4(k-1)^2-4(k+1)=0$.

$\Rightarrow (k-1)^2-(k+1)=0$. $\Rightarrow k^2-3k=0$.

$k(k-3)=0$ $\boxed{k=0}$ $\boxed{k=3}$

(i) $\boxed{(2k+1)x^2+2(k+3)x+k+5=0.}$

$a=(2k+1)$, $b=2(k+3)$, $c=(k+5)$

$\Rightarrow (2(k+3))^2-4(2k+1)(k+5)=0$

$\Rightarrow k^2+5k-4=0$

$\alpha=\frac{-b+\sqrt{D}}{2a}$, $\beta=\frac{-b-\sqrt{D}}{2a}$

$\therefore \alpha=\frac{-5+\sqrt{41}}{2}$, $\beta=\frac{-5-\sqrt{41}}{2}$

(OR) $\boxed{k=\frac{-5\pm\sqrt{41}}{2}}$

(j) $kx(x-3)=-9$

$\Rightarrow kx^2-3kx+9=0$

$D=0$, $(-3k)^2-4(k)(9)=0$.

$\Rightarrow 9k^2-36k=0$ OR $9k(k-4)=0$.

$\boxed{k=0, k=4}$

(k) $\boxed{4x^2=(3kx-1)}$

$4x^2-3kx+1=0$, $a=4$, $b=-3k$, $c=1$.

$\Rightarrow (-3k)^2-4(4)(1)=0 \Rightarrow 9k^2-16=0$

$k^2=\frac{16}{9}$, $\boxed{k=\pm\frac{4}{3}}$

Q13. Given -4 is root of $\boxed{x^2-4+ax=0}$

So put $x=-4$ in this equation

$(-4)^2-4+a(-4)=0$ $\therefore a=3$ —(1)

Given that $\boxed{x^2+ax+b=0}$ has equal roots

$D=0$, $a^2-4(1)(b)=0$. $\Rightarrow a^2=4b$

$9=4b$ OR $\boxed{b=9/4}$ —(2)

$\therefore \frac{a}{b}=3\div\frac{9}{4}$ OR $3\times\frac{4}{9}=\boxed{4/3}$

Q14. $\boxed{(a-b)x^2+(b-c)x+(c-a)=0}$

$D=0$, $(b-c)^2-4(a-b)(c-a)=0$

$(b^2+c^2-2bc)-4(ac-a^2-bc+ab)=0$

$\Rightarrow b^2+c^2-2bc-4ac+4a^2+4bc+4ab=0$

$\Rightarrow b^2+c^2+2bc+4a^2-4ac+4ab=0$.

$\Rightarrow (b+c-2a)^2=0$

using $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

$b+c-2a=0$.

OR $b+c=2a$

OR $\boxed{a=\frac{1}{2}(b+c)}$

Q15. 2 Consecutive positive integers are $x, x+1$

According to question: $x^2+(x+1)^2=1201$

$\Rightarrow x^2+x^2+1+2x=1201$

$\Rightarrow 2x^2+2x-1200=0$

$\Rightarrow \boxed{x^2+x-600=0}$

$D=b^2-4ac=(1)^2-4(1)(-600)$

$D=1+2400=2401$

$\sqrt{D}=\sqrt{2401}=49$

$\alpha=\frac{-1+49}{2}=\frac{48}{2}=24$

$\beta=\frac{-1-49}{2}=\frac{-50}{2}=-25$

OR $x=24$ & $x=-25$ — Rejected

Take $x=24$

$\therefore$ $\boxed{24 \text{ and } 25}$ are 2 integers

Q16. 2 Consecutive odd (+ve) integers $(x), (x+2)$

$\Rightarrow x^2+(x+2)^2=290$

$\Rightarrow x^2+2x-143=0$ OR $x=11, -13$

Take $x=11$, Hence $11, 13$ are odd integers

Q17. $\boxed{mx^2-14x+8=0}$

Let the roots be α and β.

$\alpha=6\beta$.

$\alpha+\beta=\frac{14}{m}$ & $\alpha\beta=8/m$

$\Rightarrow 6\beta+\beta=\frac{14}{m}$ & $6\beta\times\beta=\frac{8}{m}$

$\Rightarrow 7\beta=\frac{14}{m}$ & $6\beta^2=\frac{8}{m}$

$\Rightarrow \boxed{\beta=\frac{2}{m}}$ & $\beta^2=\frac{8}{6m}$

$\therefore \left(\frac{2}{m}\right)^2=\frac{8}{6m}$

$\Rightarrow \frac{4}{m^2}=\frac{8}{6m}$ OR $\frac{4\times 6}{8}=m$

$\boxed{m=3}$

Q18. $\boxed{x^2(a^2+b^2)-2(bd+ac)x+(c^2+d^2)=0}$

$a=(a^2+b^2)$, $b=-2(bd+ac)$, $c=(c^2+d^2)$

$b^2-4ac=0$ $[-2(bd+ac)]^2-4(a^2+b^2)(c^2+d^2)=0$

$\Rightarrow (bd+ac)^2=(a^2+b^2)(c^2+d^2)$

$\Rightarrow b^2d^2+a^2c^2+2abcd=a^2c^2+a^2d^2+b^2c^2+b^2d^2$

$\Rightarrow a^2d^2+b^2c^2-2abcd=0.$ $\Rightarrow (ad-bc)^2=0$ $\therefore \boxed{ad=bc}$

Type-4 Anyway.

Q19. $\boxed{16=-(x^2+kx)}$

$\Rightarrow x^2+kx+16=0$

$\boxed{k=\pm 8}$

Q20. $(m+5)^{0.5}+(m+12)^{0.5}=(2m+41)^{0.5}$

$\Rightarrow \sqrt{m+5}+\sqrt{m+12}=\sqrt{2m+41}$

Squaring both the sides we get

$(\sqrt{m+5}+\sqrt{m+12})^2=(\sqrt{2m+41})^2$

$(m+5)+(m+12)+2(\sqrt{m+5})(\sqrt{m+12})=2m+41$

$\Rightarrow \cancel{2m}+17+2\sqrt{m^2+17m+60}=\cancel{2m}+41$

$\Rightarrow 2\sqrt{m^2+17m+60}=24$

$\sqrt{m^2+17m+60}=12$ (Squaring Both)

$m^2+17m+60=144$

$m^2+17m-84=0.$

$(m-4)(m+21)=0$ $\therefore \boxed{m=4, -21}$

Q21. $\boxed{3x^2+5x-k=0}$

Given $\alpha-\beta=2$ —(1)

Now $\alpha+\beta=-5/3$ —(2)

Solving (1) & (2) we get

$\alpha=\frac{1}{6}$, $\beta=-\frac{11}{6}$.

Now $\alpha\beta=\frac{c}{a}=\frac{-k}{3}$

$\Rightarrow \left(\frac{1}{6}\times-\frac{11}{6}\right)=-\frac{k}{3}$

$\therefore \boxed{k=\frac{11}{12}}$

Q22. $\boxed{3y^2-10y+m=0}$

Let one root be α, $\frac{1}{\alpha}$ (other root)

$\alpha+\frac{1}{\alpha}=\frac{10}{3}$, $\alpha\times\frac{1}{\alpha}=m/3.$

$\boxed{m=3}$

Q23. $4x^2+8ax+(a+9)=0$

$a=4$, $b=8a$, $c=(a+9)$

$\alpha+\beta=\alpha\beta$ (given)

$\frac{-b}{a}=\frac{c}{a}$

OR $\frac{-8a}{\cancel{4}}=\frac{(a+9)}{\cancel{4}}$

$\boxed{a=-1}$