CODE:2002- AG-005-IIND TERM-21-22 पजियन क्रमांक REG.NO:-TMC -D/79/89/36

General Instructions:

Read the following instructions very carefully and strictly follow them :

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

EXAMINATION 2021 -22(IIND TERM)

Time : 2 Hours | Maximum Marks : 40

CLASS – XII | **MATHEMATICS**

Sr. No.	SECTION – A (6 X 2=12)	Marks allocated
Q.1	Evaluate: $\int\left(\frac{x+2}{x+4}\right)^2 e^x\ dx$. $I=\int\left(\frac{x+2}{x+4}\right)^2 e^x dx=\int e^x\left[\frac{x^2+4x+4}{(x+4)^2}\right]dx \Rightarrow I=\int e^x\left[\frac{x(x+4)}{(x+4)^2}+\frac{4}{(x+4)^2}\right]dx$ $=e^x\left[\frac{x}{x+4}+\frac{4}{(x+4)^2}\right]dx=e^x\left(\frac{x}{x+4}\right)+c$. OR Evaluate: $\int\frac{x+\sin x}{1+\cos x}dx$. ANS. $x\tan\frac{x}{2}-2\log\sec\frac{x}{2}-\log(1+\cos x)$ OR $\int\frac{x+\sin x}{1+\cos x}dx=\frac{1}{2}\int x\sec^2\frac{x}{2}dx+\int\tan\frac{x}{2}dx$	2

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	$=\frac{1}{2}\frac{x\tan\frac{x}{2}}{\frac{1}{2}}-\int\tan\frac{x}{2}dx+\int\tan\frac{x}{2}dx$	
Q.2	Solve the differential equation: $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$. $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0\Rightarrow\int\frac{dy}{\sqrt{1-y^2}}=-\int\frac{dx}{\sqrt{1-x^2}}+c$ $\Rightarrow\sin^{-1}y=-\sin^{-1}x+\sin^{-1}c$ $\Rightarrow\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]=\sin^{-1}c\Rightarrow x\sqrt{1-y^2}+y\sqrt{1-x^2}=c$.	2
Q.3	If $\vec{a}$ and $\vec{b}$ are two non-collinear unit vectors such that $\left\|\vec{a}+\vec{b}\right\|=\sqrt{3}$, find $(2\vec{a}-5\vec{b}).(3\vec{a}+\vec{b})$. Ans: $-\frac{11}{2}$	2
Q.4	Find the value of λ so that the lines $\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $\frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$ are perpendicular to each other . Ans : $\lambda=1$	2
Q.5	Consider the experiment of tossing a coin . If the coin shows tail , toss it again but if it show head , then throw a die . Find the conditional probability of the event that the die shows a number greater than 3 give that there is at least one head. $S=\{TT,TH,H1,H2,H3,H4,H5,H6\}$ Let E : Die shows a number > 3 , E : {H4, H5, H6] F : there is atleast one head. , ∴ F : {HT, H1, H2, H3, H4, H5, H6} $P(F)=\frac{3}{4}$ $P(E\cap F)=\frac{3}{12}=\frac{1}{4}$ $\therefore P(E/F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$	2
Q.6	If A and B are two independent events and $P(A)=\frac{1}{4}, P(B)=\frac{1}{2}$, Find $P(A\cup B)$. Hence find P (not A and not B). $P(A\cup B)=\frac{3}{5}\Rightarrow P(A)+P(B)-P(A\cap B)=\frac{3}{5}$ A & B are independent $\Rightarrow P(A)+P(B)-P(A)\cdot P(B)=\frac{3}{5}$ $\frac{1}{2}+p-\frac{p}{2}=\frac{3}{5}$ So $p=\frac{1}{5}$	2

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	SECTION – B (3 X 4 = 12)	
Q.7	Evaluate: $\int\left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$. $I = \int\left\{\log(\log\ x) + \frac{1}{(\log\ x)^2}\right\}dx$ Put $\log x = t$ or $x = e^t$ so that $dx = e^t dt$ $\therefore$ $I=\int\left(\log t+\frac{1}{t^2}\right)e^t dt=\int\left(\log t+\frac{1}{t}-\frac{1}{t}+\frac{1}{t^2}\right)e^t dt=\int\left(\log t+\frac{1}{t}\right)e^t dt+\int\left(-\frac{1}{t}+\frac{1}{t^2}\right)e^t dt$ $=\int \underset{II}{e^t}\ \underset{I}{\log t}\ dt+\int e^t.\frac{1}{t}dt+\int \underset{II}{e^t}\ \underset{I}{(-1/t)}\,dt+\int e^t\ \frac{1}{t^2}dt$ $=(\log t)e^t-\int\frac{1}{t}.e^t dt+\int e^t.\frac{1}{t}dt+\left(\frac{-1}{t}\right)e^t-\int\frac{1}{t^2}.e^t dt+\int e^t\frac{1}{t^2}dt+C$ $= e^t.\log\ t-\frac{1}{t}e^t + C = x\log(\log\ x)-\frac{x}{\log\ x}+C$	3
Q.8	Solve the differential equation: $(x+y)dx+xdy=0$. $(x+y)dx + xdy = 0 \implies xdy = -(x+y)dx$ $\Rightarrow \frac{dy}{dx}=-\frac{x+y}{x}$ It is homogenous equation, hence put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$, we get $v+x\frac{dv}{dx}=-\frac{x+vx}{x}=-\frac{1+v}{1}$ $\Rightarrow x\frac{dv}{dx}=-1-2v \Rightarrow \int\frac{dv}{1+2v}=-\int\frac{dx}{x}+c$ $\Rightarrow \frac{1}{2}\log(1+2v)=-\log x+\log c \Rightarrow \log\left(1+2\frac{y}{x}\right)=2\log\frac{c}{x}$ $\Rightarrow \frac{x+2y}{x}=\left(\frac{c}{x}\right)^2 \Rightarrow x^2+2xy=c$. OR Find the particular solution of the differential equation $(\tan^{-1}y-x)dy=(1+y^2)dx$, given that x = 1 when y = 0. $(1+y^2)dx-(\tan^{-1}y-x)dy=0$ $\Rightarrow \frac{dy}{dx}=\frac{1+y^2}{\tan^{-1}y-x} \Rightarrow \frac{dx}{dy}=\frac{\tan^{-1}y}{1+y^2}-\frac{x}{1+y^2}$ $\Rightarrow \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$ This is equation of the form $\frac{dx}{dy}+Px=Q$ So, I.F.	3

	$= e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}.dy} = e^{\tan^{-1} y}$. $\quad x\, e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2}\, dy$ $x\, e^{\tan^{-1}y} = \int t e^{t}\, dt = te^{t} - e^{t} + c = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$ (where $\tan^{-1}y = t$) $x = 1, y = 0 \Rightarrow c = 2 \therefore x \cdot e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + 2$ or $x = \tan^{-1}y - 1 + 2\, e^{-\tan^{-1}y}$	
Q.9	Given that vectors $\vec{a}, \vec{b}, \vec{c}$ from a triangle such that $\vec{a} = \vec{b} + \vec{c}$. Find p, q, r, s such that area of triangle is $5\sqrt{6}$ where $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$, $\vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$. $\vec{a} = \vec{b} + \vec{c} \Rightarrow p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$ $p = s + 3,\ q = 4,\ r = 2$ area $= \frac{1}{2}\lvert\vec{b} \times \vec{c}\rvert = 5\sqrt{6}$ $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix} = -10\hat{i} + (2s+12)\hat{j} + (s-9)\hat{k}$ $\therefore 100 + (2s + 12)^2 + (s - 9)^2 = (10\sqrt{6})^2 = 600$ $\Rightarrow s^2 + 6s + 55 = 0 \Rightarrow s = -11, p = -8$, or $s = 5, p = 8$	3
Q.10	Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1}, z + 1 = 0$ and $\frac{x-4}{2} = \frac{z+1}{3}, y = 0$ intersect each other. Also find their point of intersection. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda \qquad x = 2\mu + 4,\ y = 0,\ z = 3\mu - 1$ At the point of intersection $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu \qquad \lambda = 1,\ \mu = 0$ $x = 3\lambda + 1,\ y = -\lambda + 1,\ z = -1 \qquad$ so $3\lambda + 1 = 4 = 2\mu + 4$	3

	Hence the lines are intersecting Point of intersection is (4, 0, –1) OR Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the plane determined by the points P(2, 1, 2), Q(3, 1, 0) and R(4, -2, 1). Equation of line is $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$ Equation of plane is $\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0$ $\Rightarrow \quad 2x+y+z-7=0$ .... (i) general point on given line $(2\lambda+3, -3\lambda+4, 5\lambda+1)$ lies on (i) $\therefore \quad 2(2\lambda+3)+(-3\lambda+4)+(5\lambda+1)-7=0 \Rightarrow \lambda = -\frac{2}{3}$ $\therefore$ Point of intersection $\left(\frac{5}{3}, 6, -\frac{7}{3}\right)$	
	SECTION – C (4 X 4 = 16)	
Q.11	Evaluate: $\int_0^1 x(\tan^{-1} x)^2 dx$. Solution $I = \int_0^1 x(\tan^{-1} x)^2 dx$. Integrating by parts, we have $I = \frac{x^2}{2}\left[(\tan^{-1} x)^2\right]_0^1 - \frac{1}{2}\int_0^1 x^2 . 2\frac{\tan^{-1} x}{1+x^2} dx$ $= \frac{\pi^2}{32} - \int_0^1 \frac{x^2}{1+x^2} . \tan^{-1} x\, dx$ $= \frac{\pi^2}{32} - I_1$, where $I_1 = \int_0^1 \frac{x^2}{1+x^2} \tan^{-1} x dx$	4

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$$I_1 = \int_0^1 \frac{x^2+1-1}{1+x^2}\tan^{-1}x\,dx$$

$$= \int_0^1 \tan^{-1}x\,dx - \int_0^1 \frac{1}{1+x^2}\tan^{-1}x\,dx$$

$$= I_2 - \frac{1}{2}\left((\tan^{-1}x)^2\right)_0^1 \qquad = I_2 - \frac{\pi^2}{32}$$

Here $\quad I_2 = \int_0^1 \tan^{-1}x\,dx = (x\tan^{-1}x)_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx$

$$= \frac{\pi}{4} - \frac{1}{2}\left(\log\left|1+x^2\right|\right)_0^1 = \frac{\pi}{4} - \frac{1}{2}\log 2.$$

Thus $\quad I_1 = \frac{\pi}{4} - \frac{1}{2}\log 2 - \frac{\pi^2}{32}$

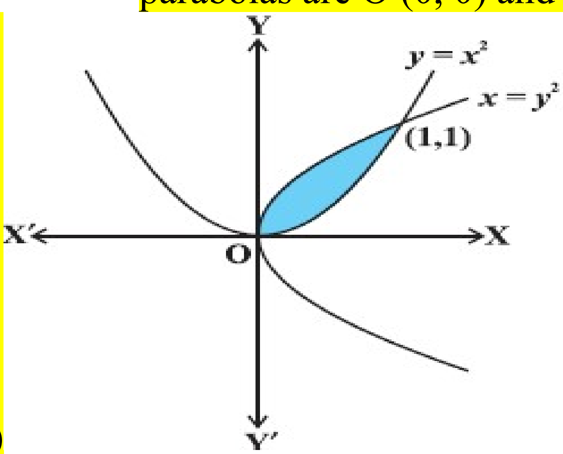
Therefore, $\quad I = \frac{\pi^2}{32} - \frac{\pi}{4} \quad \frac{1}{2}\log 2 \quad \frac{\pi^2}{32} = \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2}\log 2$

$$= \frac{\pi^2 - 4\pi}{16} + \log\sqrt{2}.$$

Ans

$\frac{\pi(\pi-4)}{16} + \frac{1}{2}\log 2$

Q.12 Find the area of the region bounded by the two parabolas $x^2 = y \,\&\, y^2 = x$. Ans : The point of intersection of these two parabolas are O (0, 0) and A (1, **4**



1) required area =

$$= \int_0^1\left[\sqrt{x} - x^2\right]dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

OR

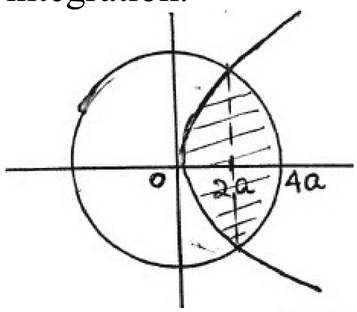
Sketch the region common to the circle $x^2+y^2 \le 16a^2$ and the parabola $y^2 \le 6ax$. Also, find the area of the region using integration.

Solving $y^2 = 6ax$ and $x^2 + y^2 = 16a^2$

we get $x^2 + 6ax - 16a^2 = 0$

$(x + 8a)(x - 2a) = 0$

$x = -8a,\ x = 2a$



$$\text{Required area} = 2\left[\int_0^{2a} \sqrt{6a}\sqrt{x}\,dx + \int_{2a}^{4a} \sqrt{16a^2-x^2}\,dx\right]$$

$$= 2\left[\left(\sqrt{6}\sqrt{a}\,\frac{2}{3}x^{3/2}\right)_0^{2a} + \left(\frac{x}{2}\sqrt{16a^2-x^2} + 8a^2\sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$$

$$= 2\left[\frac{8\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{2} - 2a^2\sqrt{3} - 8a^2\frac{\pi}{6}\right] = 2\left[\frac{2\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{3}\right] \text{sq. units}$$

Q.13 Find the equation of a plane which passes through the point (3, 2, 0) and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$. **4**

Any plane through given point is $a(x-3)+b(y-6)+c(z-4)=0$..........(i)

with $a+5b+4c=0$ ....(A)

(i) passes through (3, 2, 0) $\Rightarrow -4b-4c=0$ or $b+c=0$ ....(B)

From (A) and (B) $a+b+(4b+4c) = 0 \Rightarrow a=-b$

	$\therefore\ a = -b = c$ $\therefore$ Required eqn. of plane is $x - y + z - 1 = 0$	
Q.14	There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin? Ans : Let E1:selectionof first (biased) coin ;E2: selection of second (biased) coin ;E3: selection of third (unbiased) coin $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ Let A denote the event of getting a head Therefore, $P\left(\frac{A}{E_1}\right) = \frac{40}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{75}{100}$, $P\left(\frac{A}{E_3}\right) = \frac{1}{2}$ $P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$ $= \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{40}{100} + \frac{1}{3}\cdot\frac{75}{100} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{10}{33}$	4
	सपने वो नहीं है जो हम नींद में देखते है, सपने वो है जो हमको नींद नहीं आने देते।