Mathematics (041)

Class- XII, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section - A

[2 Marks each]

1. Find the value of $\int_{-\pi/2}^{\pi/2}(x^3+x\cos x+\tan^5 x+1)\,dx$

OR

Evaluate : $\int \frac{dx}{e^x+e^{-x}}$

2. Show that $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$ is the solution of $y=e^{-x}(A\cos x+B\sin x)$

3. Find the projection of vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on the vector $\vec{b}=2\hat{i}+2\hat{j}+\hat{k}$.

4. If the lines $\frac{x-1}{-2}=\frac{y-4}{3p}=\frac{z-3}{4}$ and $\frac{x-2}{4p}=\frac{y-5}{2}=\frac{z-1}{-7}$ are perpendicular to each other, then find the value of p.

5. If $P(A)=0.4$, $P(B)=0.8$ and $P\left(\frac{B}{A}\right)=0.6$, then $P(A\cup B)$

6. Find the probability distribution of X, the number of heads is a simultaneous toss of two coins.

Section - B

[3 Marks each]

7. Find the value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

8. Find the general solution of $\frac{dy}{dx}+y\tan x=\sec x$

OR

Solve the differential equation:

$$x\sin\left(\frac{y}{x}\right)\frac{dy}{dx}+x-y\sin\left(\frac{y}{x}\right)=0$$

Given that $x = 1$ when $y = \frac{\pi}{2}$.

9. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram.

10. Find the shortest distance between the lines:

$$\vec{r}=(t+1)\hat{i}+(2-t)\hat{j}+(1+t)\hat{k}$$

$$\vec{r}=(2s+2)\hat{i}-(1-s)\hat{j}+(2s-1)\hat{k}.$$

OR

A plane meets the co-ordinate axes at A, B and C such that the centroid of ΔABC is the point (α, β, γ). Show that the equation of the plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$.

Section - C

[4 Marks each]

11. Find : $\int \frac{\sec x}{1+\operatorname{cosec} x}dx$

12. Find the area bounded by lines $x = 2y + 3$, $y - 1 = 0$ and $y + 1 = 0$.

OR

Find the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$.

13. Find the equation of plane passing through the points $A(3, 2, 1)$, $B(4, 2, -2)$ and $C(6, 5, -1)$ and hence find the value of λ for which $A(3, 2, 1)$, $B(4, 2, -2)$, $C(6, 5, -1)$ and $D(\lambda, 5, 5)$ are coplanar.

Case-Based/Data Based

14. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. Let E_1 and E_2 be the events that selecting a student with 100% attendance and selecting a student who is not regular, respectively.

Based on the above information, answer the following questions:

(i) Find the values of $P\left(\frac{A}{E_1}\right)$ and $P\left(\frac{A}{E_2}\right)$. **[2]**

(ii) What is the probability that the student has 100% attendance. **[2]**

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