RISE OF NATION ACADEMY

Subject:- Mathematics **Time:- 90 minutes**

Min. Marks:- 20 **Chapter:- Probability, Vector, Differential equation** **Max. Marks:- 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Prove that: $\int_0^{\pi/2} \frac{\cos^{1/4} x}{\left(\sin^{1/4} x+\cos^{1/4} x\right)} dx = \frac{\pi}{4}$ **[2]**

OR

Evaluate: $\int x^3 e^x dx$

2. Write the order and degree of the differential equation $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$. **[2]**
3. For what value of' **a** the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear? **[2]**
4. Find the cartesian and vector equations of the planes through the line of intersection of the planes $\vec{r} \cdot (\hat{i} - \hat{j}) + 6 = 0$ and $\vec{r} \cdot (3\hat{i} + 3\hat{j} - 4\hat{k}) = 0$ which are at a unit distance from the origin. **[2]**
5. A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve the problem, selected at random from the book? **[2]**
6. An electronic assembly consists of two sub-systems say A and B. From previous testing procedures, the following probabilities are assumed to be known: **[2]**

   P (A fails) = 0.2

   P (B fails alone) = 0.15

   P (A and B fail) = 0.15

   Evaluate the following probabilities.

   (1) $P\left(\overline{A}|\overline{B}\right)$

   (2)P(A fails alone).

Section B

7. Evaluate: $\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$. **[3]**
8. Solve the following differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1, when x = **[3]**

OR

Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = cx$.

9. Find the value of λso that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}, -\hat{j}-\hat{k}, 3\hat{i}+\lambda\hat{j}+4\hat{k}$and $-4\hat{i}+4\hat{j}+4\hat{k}$,respectively are coplanar.

10. Find the foot of perpendicular from the point (2, 3, -8) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line.

OR

Find the distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+\hat{2}k+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

Section C

11. Evaluate: $\int\frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$ **[4]**

12. Find the area common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6ax$. **[4]**

OR

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

13. By computing the shortest distance determine whether the pairs of lines intersect or not: **[4]**
$\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}+\hat{j}-\hat{k})$

CASE-BASED/DATA-BASED

14. The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3, and the probability that he will buy a shirt given that he buys a trouser is 0.4. **[4]**

i. Find the probability that he will buy both a shirt and a trouser.

ii. Find also the probability that he will buy a trouser given that he buys a shirt.