Sample Paper 2022-23

SAMPLE PAPER 1

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = ?$ **[1]**

   a) None of these b) 2

   c) 0 d) -1

2. The equation of the line passing through the points $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is **[1]**

   a) $\vec{r} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$ - $t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$ b) $\vec{r} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$ + $\lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

   c) None Of These d) $\vec{r} = a_1(1-t)\hat{i} + a_2(1-t)\hat{j}$ + $a_3(1-t)\hat{k} + t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

3. The unit vector perpendicular to the plane passing through point $P(\hat{i} - \hat{j} + 2\hat{k}), Q(2\hat{i} - \hat{k})$ and $R(2\hat{j} + \hat{k})$ is **[1]**

   a) $\frac{1}{6}(2\hat{i} + \hat{j} + \hat{k})$ b) $2\hat{i} + \hat{j} + \hat{k}$

   c) $\frac{1}{\sqrt{6}}(2\hat{i} + \hat{j} + \hat{k})$ d) $\sqrt{6}(2\hat{i} + \hat{j} + \hat{k})$

4. If P(A) = $\frac{3}{5}$ and P(B) = $\frac{1}{5}$, find P (A ∩ B) if A and B are independent events. **[1]**

   a) $\frac{7}{25}$ b) $\frac{3}{25}$

   c) $\frac{4}{25}$ d) $\frac{8}{25}$

5. $\int \frac{dx}{\sqrt{2x-x^2}}$ =? **[1]**

   a) $\sin^{-1}(x + 1) + C$ b) $\sin^{-1}(x - 1) + C$

   c) $\sin^{-1}(x - 2) + C$ d) None of these

6. If A and B are two events such that $P(A \cup B) = \frac{5}{6}, P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$, then the events A and B are **[1]**

a) None of these
b) Independent
c) Dependent
d) Mutually exclusive

7. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2 is **[1]**

a) π
b) $\frac{\pi}{3}$
c) $\frac{\pi}{4}$
d) $\frac{\pi}{2}$

8. The direction ratios of two lines are 3, 2, -6 and 1, 2, 2 respectively. The acute angle between these lines is **[1]**

a) $\cos^{-1}\left(\frac{5}{18}\right)$
b) $\cos^{-1}\left(\frac{8}{21}\right)$
c) $\cos^{-1}\left(\frac{5}{21}\right)$
d) $\cos^{-1}\left(\frac{3}{20}\right)$

9. The length of the longer diagonal of the parallelogram constructed on $5\vec{a} + 2\vec{b}$ and $\vec{a} - 3\vec{b}$ if it is given that $|\vec{a}| = 2\sqrt{2}, |\vec{b}|$ = 3 and angle between $\vec{a}$ an $\vec{b}$ is $\frac{\pi}{4}$, is **[1]**

a) 15
b) $\sqrt{369}$
c) $\sqrt{593}$
d) $\sqrt{113}$

10. The number of arbitrary constants in the particular solution of a differential equation of second order is (are): **[1]**

a) 1
b) 3
c) 0
d) 2

11. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line x = 2 and x-axis is **[1]**

a) $\frac{8}{3}$
b) 1
c) $\frac{2}{3}$
d) $\frac{4}{3}$

12. $\int \frac{dx}{(1-e^{2x})} = ?$ **[1]**

a) $\log\left|e^{x} + \sqrt{e^{2x} - 1}\right| + C$
b) None of these
c) $-\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + C$
d) $\log\left|e^{-x} + \sqrt{e^{-2x} - 1}\right| + C$

13. $\mathrm{f(x)} = \frac{x}{\sin x}$ is **[1]**

a) Increasing in (0, 1)
b) Increasing in $\left(0, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, 1\right)$
c) None of these
d) Decreasing in (0, 1)

14. If A and B are symmetric matrices of order n (A$\neq$ B), then **[1]**

a) A + B is skew symmetric
b) A + B is a diagonal matrix
c) A + B is a zero matrix
d) A + B is symmetric

15. The value of det A where A= $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ lies in the interval **[1]**

a) [0 ,2]
b) None of these
c) [2,4]
d) (1,2)

16. If A, B are two n $\times$ n non - singular matrices, then what can you infer about AB? **[1]**

a) AB is singular

b) $(AB)^{-1}$ does not exist

c) AB is non-singular

d) $(AB)^{-1} = A^{-1}B^{-1}$

17. One branch of $\cos^{-1}$ other than the principal value branch corresponds to **[1]**

a) $[2\pi, 3\pi]$

b) $[\pi, 2\pi] - \left\{\frac{3\pi}{2}\right\}$

c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

d) $(0, \pi)$

18. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is: **[1]**

a) e^x

b) $\frac{x}{e^x}$

c) $\frac{e^x}{x}$

d) xe^x

19. **Assertion (A):** If manufacturer can sell x items at a price of ₹$\left(5 - \frac{x}{100}\right)$ each. The cost price of x items is ₹ $\left(\frac{x}{5} + 500\right)$. Then, the number of items he should sell to earn maximum profit is 240 items. **[1]**

Reason (R): The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100}$ - 300.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If A = $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, then A^{-1} = $\begin{bmatrix} 3 & -2 \\ 4 & 3 \end{bmatrix}$ **[1]**

Reason (R): A = $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$, then A^{-1} = $\begin{bmatrix} \frac{2}{13} & -\frac{5}{13} \\ \frac{3}{13} & -\frac{1}{13} \end{bmatrix}$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

22. Verify that the accompanying function is a solution: $x\frac{dy}{dx} = y$, at y =ax **[2]**

23. Solve the system of equations using Cramer's rule: **[2]**

5x - 7y + z =11, 6x - 8y - z = 15 and 3x + 2y - 6z =7.

OR

If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, then show that A - 3I =2(I + $3A^{-1}$)

24. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$, $\vec{a}$ and $\vec{b}$ **[2]**

25. A and B appear for an interview for two vacancies in the same post. The probability of A's selection is 1/6 and that of B's selection is 1/4. Find the probability that only one of them is selected. **[2]**

Section C

26. Evaluate: $\int \frac{\tan x}{(1-\sin x)} dx$. **[3]**

27. Solve the differential equation: (x + tan y) dy = sin 2y dx **[3]**

OR

Verify that the accompanying function is a solution: $\frac{d^2y}{dx^2}$ - y = 0, y(0) = 2, y'(0) = 0, at y = $e^x + e^{-x}$

28. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\overrightarrow{0}$, then prove that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ **[3]**

OR

Show that each of the given three vectors is a unit vector:

$\frac{1}{7}\left(2\hat{i}+3\hat{j}+6\hat{k}\right), \frac{1}{7}\left(6\hat{i}+2\hat{j}-3\hat{k}\right), \frac{1}{7}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$

Also, show that they are mutually perpendicular to each other.

29. Evaluate: $\int \frac{\sec x \tan x}{3 \sec x+5} dx$ **[3]**

OR

Find $\int \frac{x^2+1}{x^2-5x+6} dx$

30. Find the values of a and b so that the function f given by $f(x)=\begin{cases}1, \text{ if } x \leq 3 \\ ax+b, \text{ if } 3<x<5 \\ 7, \text{ if } x \geq 5\end{cases}$ is continuous at x = 3 and x = 5 **[3]**

31. Draw a rough sketch of the curve y = $\frac{\pi}{2}$ + 2 $\sin^2$ x and find the area between x-axis, the curve and the ordinates x = 0, x = π **[3]**

Section D

32. Solve the following LPP graphically: **[5]**

Minimize and Maximize Z = 5x + 2y

Subject to

- 2x - 3y $\leq$ - 6

x - 2y $\leq$ 2

3x + 2y $\leq$ 12

- 3x + 2y $\leq$ 3

x, y $\geq$ 0

33. Let A = R - {3}, B = R - {1]. If $f: A \rightarrow B$ be defined by $f(x)=\frac{x-2}{x-3}\ \forall x \in A$. Then, show that f is bijective. **[5]**

OR

Let A = R – {3} and B = R – {1}. Consider the function f: A $\rightarrow$ B defined by $f(x)=\left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

34. Show that the lines $\vec{r}=(2\hat{i}-3\hat{k})+\lambda(\hat{i}+2\hat{j}+3\hat{k})$ and $\vec{r}=(2\hat{i}+6\hat{j}+3\hat{k})+\mu(2\hat{i}+3\hat{j}+4\hat{k})$ intersect. Also, find their point intersection. **[5]**

OR

Find the shortest distance between the lines $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$.

35. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}(2x\sqrt{1-x^2})$, if $-\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

(i) Express cost of tank as a function of height(h).

(ii) Verify by second derivative test that cost is minimum at critical point.

(iii) Find the value of h at which c(h) is minimum.

OR

Find the minimum cost of tank?

37. **Read the text carefully and answer the questions:** **[4]**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association). Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

(i) Represent the given information in matrix algebra.

(ii) If ₹15000 is invested in bond X, then find total amount of interest received on both bonds?

(iii) If the trust fund obtains an annual total interest of ₹ 3200, then find the investment in two bonds.

OR

If the amount of interest given to old age home is ₹500, then find the amount of investment in bond Y.

38. **Read the text carefully and answer the questions:** **[4]**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) Find the probability that Ajay gets Grade A in all subjects.

(ii) Find the probability that he gets Grade A in no subjects.