CODE:2101- AG-2-FC-TS-22-23 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

EXAMINATION 2022 -23

Time : 3 Hours — Maximum Marks : 80

CLASS – XII — **MATHEMATICS**

Sr. No.	**SECTION – A** **This section comprises of very short answer type-questions (VSA) of 2 marks each**	Marks allocated
Q.1	If $A=\begin{bmatrix}-1 & 2\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}3\\1\end{bmatrix}$, $AX=B$, then $X=$ (a) [5 7] (b) $\frac{1}{3}\begin{bmatrix}5\\7\end{bmatrix}$ (c) $\frac{1}{3}[5\ 7]$ (d) $\begin{bmatrix}5\\7\end{bmatrix}$	1
Q.2	If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, then (a) $x^2+y^2+z^2+xyz=0$ (b) $x^2+y^2+z^2+2xyz=0$ (c) $x^2+y^2+z^2+xyz=1$ (d) $x^2+y^2+z^2+2xyz=1$	1

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Q.3	If $\lvert \mathbf{a} \rvert = 3,\ \lvert \mathbf{b} \rvert = 4$ and $\lvert \mathbf{a}+\mathbf{b} \rvert = 5$, then $\lvert \mathbf{a}-\mathbf{b} \rvert =$ (a) 6 (b) 5 (c) 4 (d) 3	1
Q.4	If function $f(x) = \lvert x-3 \rvert + \lvert x-4 \rvert$, then which statement is true **(a)** f(x) is differentiable at x = 3 **(b)** f(x) is differentiable at x = 4 **(c)** f(x) is not differentiable at x =3 & 4 **(d)** none	1
Q.5	$\int \frac{x}{1+x^4}\,dx =$ (a) $\frac{1}{2}\cot^{-1}x^2+c$ (b) $\frac{1}{2}\tan^{-1}x^2+c$ (c) $\cot^{-1}x^2+c$ (d) $\tan^{-1}x^2+c$	1
Q.6	Order and degree of differential equation $\frac{d^2y}{dx^2} = \left\{ y + \left(\frac{dy}{dx}\right)^2 \right\}^{1/4}$ are (a) 4 and 2 (b) 1 and 2 (c) 1 and 4 (d) 2 and 4	1
Q.7	The minimum value of the objective function Z = 2x + 10y for linear constraints $x - y \geq 0$, $x - 5y \leq -5$ and $x, y \geq 0$ is (a) 10 (b) 15 (c) 12 (d) 8	1
Q.8	If $\mathbf{a} = \mathbf{i}+2\mathbf{j}+3\mathbf{k}$, $\mathbf{b} = -\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{c} = 3\mathbf{i}+\mathbf{j}$, then the unit vector along its resultant is (a) $3\mathbf{i}+5\mathbf{j}+4\mathbf{k}$ (b) $\frac{3\mathbf{i}+5\mathbf{j}+4\mathbf{k}}{50}$ (c) $\frac{3\mathbf{i}+5\mathbf{j}+4\mathbf{k}}{5\sqrt{2}}$ (d) None of these	1
Q.9	The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx =$ (a) 2 (b) $\frac{3}{4}$ (c) 0 (d) None of these	1
Q.10	Sum of all element of $\begin{bmatrix} -6 & 5 \\ -7 & 6 \end{bmatrix}^{-1} =$ (a) -2 (b) 2 (c) 24 (d) NONE	1
Q.11	Perpendicular distance of the point (3, 4, 5) from the y-axis, is (a) $\sqrt{34}$ (b) $\sqrt{41}$ (c) 4 (d) 5	1
Q.12	If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then x is (a) $\frac{13}{25}$ (b) $-\frac{25}{13}$ (c) $\frac{5}{13}$ (d) $\frac{25}{13}$	1
Q.13	If $AB = C$, then matrices A, B, C are (a) $A_{2\times3}, B_{3\times2}, C_{2\times3}$ (b) $A_{3\times2}, B_{2\times3}, C_{3\times2}$ (c) $A_{3\times3}, B_{2\times3}, C_{3\times3}$ (d) $A_{3\times2}, B_{2\times3}, C_{3\times3}$	1

Q.14	The probability that a man will be alive in 20 years is $\frac{3}{5}$ and the probability that his wife will be alive in 20 years is $\frac{2}{3}$. Then the probability that at least one will be alive in 20 years, is (a) $\frac{13}{15}$ (b) $\frac{7}{15}$ (c) $\frac{4}{15}$ (d) None of these	1
Q.15	The solution of the differential equation $\frac{dy}{dx} = (1+x)(1+y^2)$ is (a) $y = \tan(x^2 + x + c)$ (b) $y = \tan(2x^2 + x + c)$ (c) $y = \tan(x^2 - x + c)$ (d) $y = \tan\left(\frac{x^2}{2} + x + c\right)$	1
Q.16	$\frac{d}{dx}\left(\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}\right)$ is equal to (a) $\frac{1}{1+x^2}$ (b) $\frac{1}{2(1+x^2)}$ (c) $\frac{x^2}{2\sqrt{1+x^2}(\sqrt{1+x^2}-1)}$ (d) $\frac{2}{1+x^2}$	1
Q.17	If $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, then the projection of **b** on **a** is (a) 3 (b) 4(c) 5 (d) 6	1
Q.18	The direction cosines of the line $\frac{3x+1}{-3} = \frac{3y+2}{6} = \frac{z}{-1}$ are (a) $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$ (b) $\left(-1, \frac{2}{3}, 1\right)$ (c) $\left(-\frac{1}{2}, 1, -\frac{1}{2}\right)$ (d) $\left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$	1
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
Q.19	**Assertion (A) :** minor of an element of a determinant of order $n(n \geq 2)$ is a determinant of order n . **Reason (R) :** If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to $\frac{1}{\|A\|}$	1
Q.20	**Assertion (A) :** The maximum value of Z = x+3y. such that $2x + y \leq 20$, $x + 2y \leq 20$, x, $y \geq 0$ is 30. **Reason (R) :** The variables that enter into the problem are called decision variables.	1
	SECTION – B **This section comprises of very short answer type-questions (VSA) of 2 marks each**	

Q.21	A 13 m -long ladder is leaning against a wall . The bottom of the ladder is pulled along the ground ,away from the wall ,at the rate of 2 m/sec . How fast is the height on the wall decreasing when the foot of the ladder is 5m away from the wall .	2
Q.22	Find the values of 'a' for which the vector $\vec{r} = (a^2-4)i + 2j - (a^2-9)k$ makes acute angles with the coordinate axes.	2
Q.23	If $4\sin^{-1}x + \cos^{-1}x = \pi$ then find the value of x . OR Let $f: N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if n is odd} \\ \frac{n}{2}, & \text{if n is even} \end{cases}$ for all n $\in$ N. Find whether the function f is bijective.	2
Q.24	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$,show that $\sqrt{1-x^2}\frac{dy}{dx} = \sqrt{1-y^2}$.	2
Q.25	Find the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$. OR Find the vector and Cartesian equations of a line through the point (1, -1, 1) and perpendicular to the lines joining the points (4, 3, 2), (1, -1, 0) and (1, 2, -1), (2, 1, 1).	2
	SECTION – C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
Q.26	Bag A contains 3 red and 5 black balls, while bag B has 2 red and 3 black balls. One ball is drawn from bag A and two from bag B at random at a time. Find the probability that out of the three balls drawn, two are black and one is red. OR There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin?	3
Q.27	Evaluate: $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$.	3
Q.28	Evaluate: $\int_0^1 \|3x^2-1\|\,dx$. OR	3

	Evaluate: $\int_0^{\pi}\frac{x}{a^2\cos^2 x+b^2\sin^2 x}dx.$	
Q.29	Evaluate: $\int\frac{1}{2e^{2x}+3e^{x}+1}dx$.	3
Q.30	Find the intervals in which $f(x)=x^3-12x^2+36x+17$ is increasing or decreasing .	3
Q.31	Find the particular solution of the differential equation $(y-\sin x)dx+(\tan x)dy=0$ satisfying the condition that y = 0 when x = 0. OR The slope of the tangent to a curve at any point (x, y)on it is given by $\frac{y}{x}-\cot\frac{y}{x}.\cos\frac{y}{x},(x\rangle 0,y\rangle 0)$ and the curve passes through the point $(1,\pi/4)$. Find the equation of the curve .	3
	SECTION – D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
Q.32	Check whether the relation R on R defined as R = {(a, b): $a\le b^3$} is reflexive , symmetric or transitive. OR Consider $f:R_+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is invertible with $f^{-1}(y)=\left[\frac{\sqrt{y+6}-1}{3}\right]$. Also find $(i)f^{-1}(43)(ii)f^{-1}(163)$ (iii) $f^{-1}(10)$ (iv) y if $f^{-1}(y)=\frac{4}{3}$, .	5
Q.33	Using integration, find the area of the triangle bounded by the lines 11 = 7x - 2y , 19 = 3x + 2y and x – y = 3.	5
Q.34	Find the vector equation of the line parallel to the line $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{4}$ and passing through the point (2 , 4, 5) . Also find the distance between two lines . OR Find the equations of the line through A (5 ,-3 , 2)and through the intersection of the lines $\frac{x-2}{1}=\frac{y-3}{5}=\frac{z-4}{4}$ & $\frac{x-4}{3}=\frac{y-2}{4}=\frac{z+3}{-3}$.	5
Q.35	A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas machine III must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three	5

machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

SECTION – E

(This section comprises of 3 case study / passage – based questions of 4 marks each with two sub parts (i),(ii),(iii) of marks 1, 1, 2 respectively.The third case study question has two sub – parts of 2 marks each.)

Q.36

Case Study based-1

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats and plates from recycled material at a cost of 25 rs, 100 rs and 50 rs each. The number of articles sold by school A, B, C are given below.

Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

i. Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in rs) of given products per unit, in matrix form. 1

ii. Find the funds collected by school A, B and C by selling the given articles. 1

iii. If they increase the cost price of each unit by 20%, then write the matrix representing new price. 2

OR

Find the total funds collected for the required purpose after 20% hike in price.

Q.37

Case Study based-2

On the request of villagers, a construction agency design a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown

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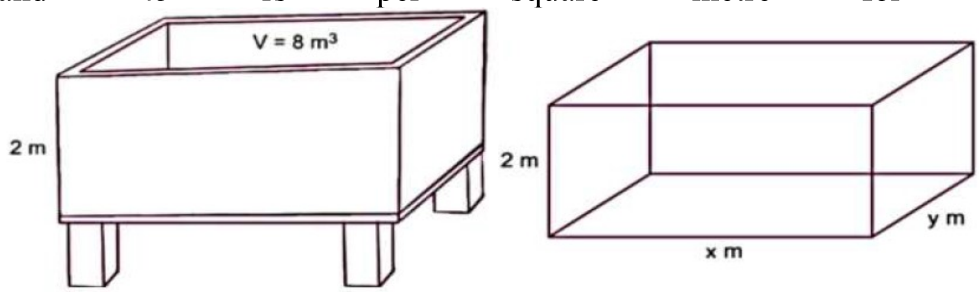
	below. The construction of the tank costs 70 rs per sq. metre for the base and 45 rs per square metre for sides. 	
i.	Express making cost C in terms of length of rectangle base.	1
ii.	If x and y represent the length and breadth of its rectangular base, then find the relation between the variables.	1
iii.	Find the value of x so that the cost of construction is minimum. OR Verify by second derivative test that cost is minimum at a critical point.	2
Q.38	**Case Study based-3** Mr. Ajay is taking up subjects of mathematics, physics and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3 and 0.5  respectively.	
i.	Find the probability that Ajay gets Grade A in all subjects.	2
ii.	Find the probability that he gets Grade A in no subjects.	2
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	"समय और शिक्षा का सही उपयोग ही व्यक्ति को सफल बनाता है।"	