CODE:1410- AG-TS-22-23

पजियन क्रमांक REG.NO:-TMC -D/79/89/36

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

EXAMINATION 2022 -23

Time : 3 Hours — Maximum Marks : 80

CLASS – XII — **MATHEMATICS**

Sr. No.	**SECTION – A** **This section comprises of very short answer type-questions (VSA) of 2 marks each**	Marks allocated
Q.1	The matrix $\begin{bmatrix} \lambda & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is invertible, if (a) $\lambda \neq -15$ (b) $\lambda \neq -17$ (c) $\lambda \neq -16$ (d) $\lambda \neq -18$	1
Q.2	$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) =$ $(a)\,\pi/2\,(b)\,\pi/3\,(c)\,\pi/4\,(d)\,-3\pi/4$	1

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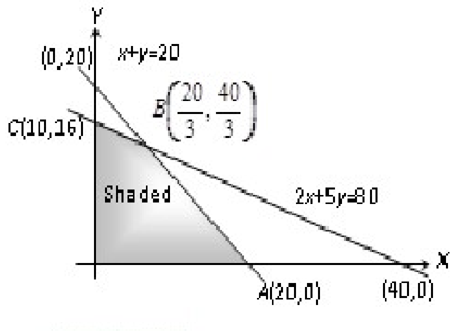
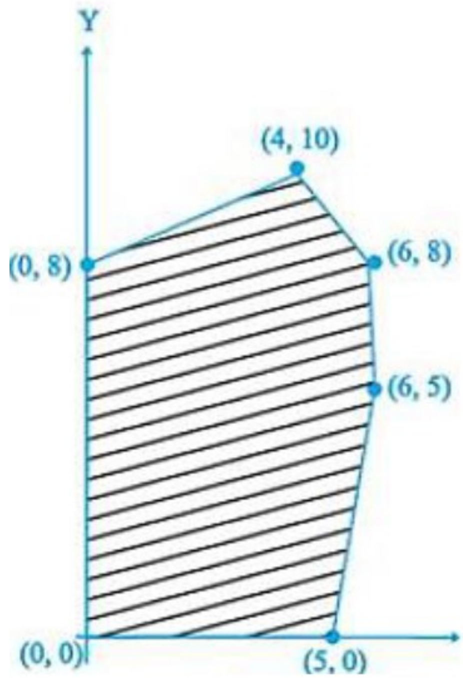
Q.3	A unit vector perpendicular to the plane determined by the points (1, – 1, 2), (2, 0, – 1) and (0, 2, 1) is (a) $\pm\frac{1}{\sqrt{6}}(2\mathbf{i}+\mathbf{j}+\mathbf{k})$ (b) $\frac{1}{\sqrt{6}}(\mathbf{i}+2\mathbf{j}+\mathbf{k})$ (c) $\frac{1}{\sqrt{6}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$ (d) $\frac{1}{\sqrt{6}}(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	1
Q.4	f is defined as following : $f(x)=\begin{Bmatrix} \lvert x\rvert+3 & if x\le -3 \\ -2x & -3<x<3 \\ 6x+2 & if x\ge 3\end{Bmatrix}$ all point of discontinuity of f , select most suitable option (a) $f(x)$ is discontinuous at x = 3 (b) $f(x)$ is continuous at x = 3 (c) $f(x)$ is discontinuous at x = -3 (d) $f(x)$ is continuous at x = -3 & $f(x)$ is discontinuous at x = 3	1
Q.5	$\int e^{x}\left[f(x)+f'(x)\right]dx$ is equal to (a) $e^{x}f(x)$ (b) e^{x} (c) $e^{x}f'(x)$ (d) None of these	1
Q.6	The equation of the curve which passes through the point (1, 1) and whose slope is given by $\frac{2y}{x}$, is (a) $y=x^2$ (b) $x^2-y^2=0$ (c) $2x^2+y^2=3$ (d) None of these	1
Q.7	The maximum value of $\mu=3x+4y$, subject to the conditions $x+y\le 40, x+2y\le 60, x,y\ge 0$ is (a) 130 (b) 120 (c) 40 (d) 140	1
Q.8	If $\lvert\mathbf{a}\rvert=\lvert\mathbf{b}\rvert=1$ and $\lvert\mathbf{a}+\mathbf{b}\rvert=\sqrt{3}$, then the value of $(3\mathbf{a}-4\mathbf{b}).(2\mathbf{a}+5\mathbf{b})$ is (a) – 21 (b) –21/2 (c) 21 (d) 21/2	1
Q.9	$\int_{-1}^{1}\frac{x^3+\lvert x\rvert+1}{x^2+2\lvert x\rvert+1}dx=$ (a) log 2 (b) 2log 2 (c) - log 2 (d) none of these	1
Q.10	If A and B are square matrices of order 3 such that $\lvert A\rvert=-1,\ \lvert B\rvert=3$, then $\lvert 3AB\rvert=$ (a) – 9 (b) – 81 (c) – 27 (d) 81	1

Q.11	Shaded region is represented by  (a) $2x+5y \geq 80,\ x+y \leq 20,\ x \geq 0,\ y \leq 0$ (b) $2x+5y \geq 80,\ x+y \geq 20,\ x \geq 0,\ y \geq 0$ (c) $2x+5y \leq 80,\ x+y \leq 20,\ x \geq 0,\ y \geq 0$ (d) $2x+5y \leq 80,\ x+y \leq 20,\ x \leq 0,\ y \leq 0$	1
Q.12	If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ and $A\ adj\ A=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then k is equal to (a) 0 (b) 1 (c) $\sin\alpha\cos\alpha$ (d) $\cos 2\alpha$	1
Q.13	For any 2×2 matrix A, if $A(adj\ A)=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}$ then $\lvert A \rvert$ is equal (a) 0 (b) 10 (c) 20 (d) 100	1
Q.14	A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is 1/4 and that of the woman's selection is 1/3. What is the probability that none of them will be selected (a) $\frac{1}{2}$ (b) $\frac{1}{12}$ (c) $\frac{1}{4}$ (d)None of these	1
Q.15	The solution of the differential equation $(1+x^2)\frac{dy}{dx}=x$ is (a) $y=\tan^{-1}x+c$ (b) $y=-\tan^{-1}x+c$ (c) $y=\frac{1}{2}\log_e(1+x^2)+c$ (d) $y=-\frac{1}{2}\log_e(1+x^2)+c$	1
Q.16	If $y=\sin^{-1}\frac{2x}{1+x^2}+\sec^{-1}\frac{1+x^2}{1-x^2}$, then $\frac{dy}{dx}=$ (a) $\frac{4}{1-x^2}$ (b) $\frac{1}{1+x^2}$ (c) $\frac{4}{1+x^2}$ (d) $\frac{-4}{1+x^2}$	1
Q.17	If **a**, **b**, **c** are mutually perpendicular vectors of equal magnitudes, then the angle between the vectors **a** and $\mathbf{a}+\mathbf{b}+\mathbf{c}$ is	1

	(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\cos^{-1}\frac{1}{\sqrt{3}}$ (d) $\frac{\pi}{2}$	
Q.18	In the given graph, the feasible region for a LPP is shaded. The objective function Z = 2x – 3y, will be minimum at:  a) (4, 10) b) (6, 8) c) (0, 8) d) (6, 5)	1
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
Q.19	Assertion (A): $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where A_{ij} is cofactor of a_{ij} Reason (R): $\Delta =$ Sum of the products of elements of any row (or coloumn) with their correspounding cofactors .	1
Q.20	Assertion (A): The acute angle between the line $\vec{r} = i + j + 2k + \lambda(i - j)$ and the x-axis $\frac{\pi}{4}$. Reason (R): The acute angle θ between the lines $\vec{r} = x_1 i + y_1 j + z_1 k + \lambda(a_1 i + b_1 j + c_1 k)$ & $\vec{r} = x_2 i + y_2 j + z_2 k + \mu(a_2 i + b_2 j + c_{21} k)$ is given by $\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$	1

	SECTION – B **This section comprises of very short answer type-questions (VSA) of 2 marks each**	
Q.21	A man 160cm tall , walks away from a source of light situated at the top of a pole 6 meter high at the rate of 1.1 m/s . How fast is the length of his shadow increasing when he is 1 meter away from the pole ?	2
Q.22	If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find the value of $\lvert 2\hat{a}+\hat{b}+\hat{c} \rvert$.	2
Q.23	Find the value of : $2\sin^{-1}\frac{1}{2}+3\tan^{-1}(-1)+2\cos^{-1}\left(-\frac{1}{2}\right)+4\sec^{-1}(\sqrt{2})$. OR Show that function $f : R \to \{x \in R : -1 < x < 1\}$ defined by $f(x)=\frac{x}{1+\lvert x\rvert}, x \in R$,is one-one & onto function .	2
Q.24	If $x^{y} = e^{x-y}$, then Prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$.	2
Q.25	Find the point on the line : (x + 2)/3 = (y + 1)/2 = (z – 3)/2 at a distance $3\sqrt{2}$ from the point (1, 2, 3). OR A line passing through the point a with position vector $\vec{a}=4\hat{i}+2\hat{j}+2\hat{k}$ is parallel to the vector $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$. Find the length of the perpendicular drawn on this line from a point P position vector $\vec{r}_1=\hat{i}+2\hat{j}+3\hat{k}$.	2
	SECTION – C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
Q.26	In a hostel, 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A student is selected at random. (a) Find the probability that she reads neither Hindi nor English news papers. (b) If she reads Hindi news paper, find the probability that she reads English news paper. (c) If she reads English news paper, find the probability that she reads Hindi news paper. OR There are 4 cards numbered 1,3,5 and 7, one number of one card. Two cards are drawn at random without replacement. Let X denote the sum of the number on the two drawn cards. Find the mean and variance of X.	3

Q.27	Evaluate: $\int \frac{(2x+1)dx}{(4-3x-x^2)}$	3
Q.28	Evaluate: $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$ OR Evaluate: $\int_{0}^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx.$	3
Q.29	Evaluate: $\int \frac{dx}{(x+1)^{1/3} + (x+1)^{1/2}}$	3
Q.30	Find the intervals in which $f(x) = (x-1)^3 (x-2)^2$ is increasing or decreasing .	3
Q.31	Find the particular solution of the differential equation $\frac{dy}{dx} + y \tan x = 3x^2 + x^3 \tan x, x \neq \frac{\pi}{2}$, given that y = 0 when $x = \frac{\pi}{3}$. OR Show that the differential equation :x dy –y dx = $\sqrt{x^2+y^2}$ dx is homogeneous, and solve it.	3
	SECTION – D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
Q.32	Test whether relation R defined on R as $R = \{(a,b) : a^2 - 4ab + 3b^2 = 0; a, b \in R\}$ is reflexive symmetric and transitive . OR Consider $f : R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Show that f is invertible with $f^{-1}(y) = \left[\frac{\sqrt{5y+54} - 3}{5}\right]$.	5
Q.33	Find the area cut off the parabola $4y = 3x^2$ by the straight line $2y = 3x + 12$.	5
Q.34	A line makes angle $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. OR Find the foot of perpendicular from the point (2, 3, 4) to the line	

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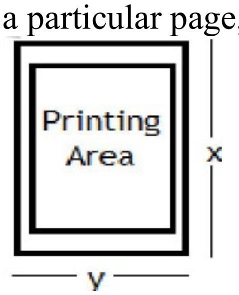
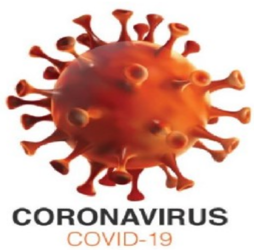
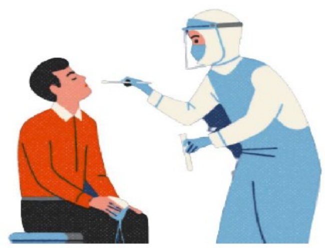
	$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also find the perpendicular distance from the given point to the line.	
Q.35	If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$, find A^{-1} and use it to solve the system of equations: x + y + 2z = 0; x + 2y – z = 9; x – 3y + 3z = - 14 .	5
	SECTION – E (This section comprises of 3 case study / passage – based questions of 4 marks each with two sub parts (i),(ii),(iii) of marks 1, 1, 2 respectively.The third case study question has two sub – parts of 2 marks each.)	
Q.36	**Case Study based-1** A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.	
i.	What number of rackets and bats must be made if the factory is to work at full capacity?	2
ii.	If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.	2
Q.37	**CASE STUDY-2** There is a local printing press, whose owner is given a bulk order for printing of a magazine by a school of the same locality. He shows variety of pages to school administration. Following is the pictorial description for a particular page, selected by school administration.  The total area of the page is 150 cm^2 . The combined width of the margin at the top and bottom is 3 cm and the side 2 cm. Using the information given above, answer the following :	
i.	The relation between x and y is given by (a) (x - 3)y 150 = (b) xy =150 (c) x(y - 2) =150 (d) (x - 2)(y - 3) = 150	1
ii.	The area of the printable region of the page, in terms of x, is (a)$156+2x+\frac{450}{x}$ (b)$156-2x+3\left(\frac{150}{x}\right)(c)156-2x-15\left(\frac{3}{x}\right)$(d) $156-2x+-3\left(\frac{150}{x}\right)$	1

iii.	For what value of ‘x’, the printable area of the page is maximum? (a) 15 cm (b) 10 cm (c) 12 cm (d) 15 units OR What should be dimension of the page so that it has maximum area to be printed? (a) Length = 1 cm, width =15 cm (b) Length =15 cm, width =10 cm (c) Length =15 cm, width =12 cm (d) Length 150 cm, width 1 cm	2
Q.38	**Case Study based-3** Testing is very important during the Covid-19 pandemic. By examining the test done at a government hospital, the probability that Covid-19 is detected when a person is actually suffering is 0.99. The probability that the doctor diagnosis incorrectly that a person has Covid-19 on the basis of test is 0.001. In a metro city, 1 in 1000 suffers from Covid-19, a person is selected at random and is diagnosed to have Covid-19.   Based on the above information answer the following :	
i.	The probability of a person diagnosed with Covid-19, is (a) 0.1989 (b) 0.00099 (c) 0.01989 (d) 0.001989	2
ii.	The probability that the doctor diagnosis correctly that a selected person has Covid-19 on the basis of test, is (a) 99 /100 (b) 1 /100 (c) 999 /1000 (d) 1 /1000	2
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	"शिक्षा भविष्य का पासपोर्ट है, कल के लिए जो आज इसकी तैयारी करते हैं।"	

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