CODE:2801- AG-3-FC-TS-22-23 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

EXAMINATION 2022 -23

Time : 3 Hours — Maximum Marks : 80

CLASS – XII — **MATHEMATICS**

Sr. No.	**SECTION – A** **This section comprises of very short answer type-questions (VSA) of 2 marks each**	Marks allocated
Q.1	The matrix $\begin{bmatrix} 2 & \lambda & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ is non singular, if (a) $\lambda \neq -2$ (b) $\lambda \neq 2$ (c) $\lambda \neq 3$ (d) $\lambda \neq -3$	1
Q.2	The shortest distance of the point (a, b, c) from the x-axis is (a) $\sqrt{(a^2+b^2)}$ (b) $\sqrt{(b^2+c^2)}$ (c) $\sqrt{(c^2+a^2)}$ (d) $\sqrt{(a^2+b^2+c^2)}$	1

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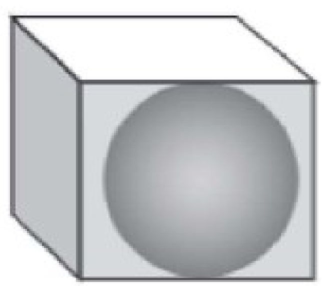
Q.3	If $\vec{a}.\vec{b}=\vec{a}.\vec{c}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq\mathbf{0}$, then (a) $\vec{b}=\mathbf{0}$ (b) $\vec{b}\neq\vec{c}$ (c) $\vec{b}=\vec{c}$ (d) None of these	1
Q.4	If $f(x)=\begin{cases}x^2+3x+a, for x\le 1\\ bx+2, for x>1\end{cases}$ is everywhere differentiable, find the values of a and b **(a)** $a=3, b=-5$ **(b)** $a=3, b=5$ **(c)** $a=-3, b=5$ **(d)** none	1
Q.5	$\int \frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx =$ (a) $e\,.\,3^{-3x}+c$ (b) $e^3\log x+c$ (c) $\frac{x^3}{3}+c$ (d)None of these	1
Q.6	Which of the following differential equations has the same order and degree (a) $\frac{d^4y}{dx^4}+8\left(\frac{dy}{dx}\right)^6+5y=e^x$ (b) $5\left(\frac{d^3y}{dx^3}\right)^4+8\left(1+\frac{dy}{dx}\right)^2+5y=x^8$ (c) $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{2/3}=4\frac{d^3y}{dx^3}$ (d) $y=x^2\frac{dy}{dx}+\sqrt{1+\left(\frac{dy}{dx}\right)^2}$	1
Q.7	A linear programming problem is as follows: $Minimize\ Z=30x+50y$ subject to the constraints, $3x+5y\ge 15$ $2x+3y\le 18$ $x\ge 0, y\ge 0$ In the feasible region, the minimum value of Z occurs at a) a unique point b) no point c) infinitely many points d) two points only	1
Q.8	If $\vert\vec{\mathbf{a}}.\vec{b}\vert=3$ and $\vert\vec{\mathbf{a}}\times\vec{b}\vert=4$, then the angle between $\vec{\mathbf{a}}$ and $\vec{b}$ is (a) $\cos^{-1}\frac{3}{4}$ (b) $\cos^{-1}\frac{3}{5}$ (c) $\cos^{-1}\frac{4}{5}$ (d) $\frac{\pi}{4}$	1
Q.9	If $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}\,dx=k\pi$, then $k=$ (a) -a (b) -2a (c) 2a (d) a	1
Q.10	The Cofactor of $\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}$ is	1

	(a) $\begin{bmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$ (d)None of these	
Q.11	Maximize z = 3x + 2y, subject to $x + y \geq 1$, $y - 5x \leq 0$, $x - y \geq -1$, $x + y \leq 6$, $x \leq 3$ and $x, y \geq 0$ (a) x = 3 (b) y = 3 (c) z = 15 (d) All the above	1
Q.12	For any square matrix A, AA^T is a (a) Unit matrix (b)Symmetric matrix (c) Skew symmetric matrix (d)Diagonal matrix	1
Q.13	If $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$, then the value of $\lvert Coff\ A \rvert$ is (a) 36 (b)72 (c)144 (d)None of these	1
Q.14	If A and B are two independent events such that $P(A \cap B') = \frac{3}{25}$ and $P(A' \cap B) = \frac{8}{25}$, then $P(A) =$ (a) $\frac{1}{5}$ (b) $\frac{3}{8}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$	1
Q.15	The solution of the differential equation $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ is (a) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$ (b) $\tan^{-1}\left(\frac{y}{x}\right) = -\log x + c$ (c) $\sin^{-1}\left(\frac{y}{x}\right) = \log x + c$ (d) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$	1
Q.16	$\frac{d}{dx}\left[\sin^2 \cot^{-1}\left\{\sqrt{\frac{1-x}{1+x}}\right\}\right]$ equals (a) -1 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1	1
Q.17	If *ABCD* is a parallelogram and the position vectors of *A, B, C* are $\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $7\mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$, then the position vector of *D* will be (a) $7\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ (b) $7\mathbf{i} + 9\mathbf{j} + 11\mathbf{k}$ (c) $9\mathbf{i} + 11\mathbf{j} + 13\mathbf{k}$ (d) $8\mathbf{i} + 8\mathbf{j} + 8\mathbf{k}$	1
Q.18	A line makes angles of $45°$ and $60°$ with the positive axes of *X* and *Y* respectively. The angle made by the same line with the positive axis of *Z*, is (a) $30°$ or $60°$ (b) $60°$ or $90°$ (c) $90°$ or $120°$ (d) $60°$ or $120°$	1
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a	

	statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
Q.19	**Assertion (A) :** $f(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing outside the interval (1, 2). **Reason (R) :** $f'(x) < 0$ for $x \in (1, 2)$	1
Q.20	**Assertion (A) :** The equation of the line joining A(1, 3) and B(0, 0) is given by y = 3x. **Reason (R) :** The area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) in the form of determinant is $\triangle = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$	1
	SECTION – B **This section comprises of very short answer type-questions (VSA) of 2 marks each**	
Q.21	A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.	2
Q.22	Find the projection of $\vec{b} + \vec{c}$ on $\vec{a}$ where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.	2
Q.23	Prove that : $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$ OR Prove that g is Bijection function $g(x) = \frac{4x+3}{3x+4}$. & $g: R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$. Find the inverse of g hence find $g^{-1}(0)$ and x such that $g^{-1}(x) = 2$.	2
Q.24	Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$. with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$.	2
Q.25	If line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k and hence. OR Find the equation of a line passing through the point (1, 2, -4) and perpendicular to two lines $\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.	2

	SECTION – C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
Q.26	A bag contains 5 white, 7 red and 8 black balls. If 4 balls are drawn one by one with replacement, what is the probability that (i) none of white (ii) all are whites .(iii) at least one white . OR Out of a group of 8 highly qualified doctors in a hospital, 6 are very kind and cooperative with their patients and so are very popular, while the other two remain reserved. For a health camp, three doctors are selected at random. Find the probability distribution of the number of very popular doctors.	3
Q.27	Evaluate : $\int \frac{(\sin x - x\cos x)dx}{x(x+\sin x)}$.	3
Q.28	Evaluate : $\int_0^1 \cot^{-1}(1-x+x^2)dx$. OR Evaluate: $\int_2^4 \{\lvert x-2\rvert+\lvert x-3\rvert+\lvert x-4\rvert\}dx$.	3
Q.29	Evaluate: $\int \frac{x^2 dx}{x^2+6x-3}$	3
Q.30	Solve the following linear programming problem graphically: Minimize Z = 6x + 3y .Subject to $4x+y \ge 80, x+5y \ge 115, 3x+2y \le 150, x \ge 0, y \ge 0$.	3
Q.31	Solve the differential equation : $ye^y dx = (y^3 + 2xe^y)dy$. OR Solve the following differential equation: $\sqrt{1+x^2+y^2+x^2y^2} + x\,y\frac{dy}{dx} = 0$	3
	SECTION – D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
Q.32	Determine whether the relation R defined on the set R of all real number as R = {(a,b); a,b ∈ R and $a-b+\sqrt{3} \in S$, where S is the set of all irrational numbers}, is reflexive, symmetric and transitive. OR Let A = {1, 2, 3…… 9} and R be the relation in $A \times A$ defined by (a, b) R(c, d) if a + d = b + c for (a, b), (c, d) $\in A \times A$. . Prove that R is an	5

	equivalence relation and also obtain the equivalence class [(2, 5)] .	
Q.33	Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bounded by x = 0, x = 4 , y = 4, y = 0 into three equal parts.	5
Q.34	Find the points on the lines $\frac{x-6}{3} = -(y-7) = (z-4)$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ which are nearest to each other. Hence find the shortest distance between the given lines. OR Find the equation of the line drawn through point (1, 0, 2) to meet at right angles the line $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$.	5
Q.35	Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix}$,find the inverse of A and hence solve the following matrix equation $XA = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$.	5
	SECTION – E (This section comprises of 3 case study / passage – based questions of 4 marks each with two sub parts (i),(ii),(iii) of marks 1, 1, 2 respectively.The third case study question has two sub – parts of 2 marks each.)	
Q.36	**Case Study based-1** Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$ is	
i.	Increasing	2
ii.	Decreasing	2
Q.37	**CASE STUDY-2** Shreya got a rectangular parallelopiped shaped box and spherical ball inside it as return gift. Sides of the box are x, $2x$ and $x/3$, while radius of the ball is r.  Based on the above information, answer the following questions.	
i.	If S represents the sum of volume of parallelopiped and sphere, then S can be written as	1

	(a) $\frac{4x^3}{3}+\frac{2}{2}\pi r^2$ (b) $\frac{2x^2}{3}+\frac{4}{3}\pi r^2$ (c) $\frac{2x^3}{3}+\frac{4}{3}\pi r^3$ (d) $\frac{2}{3}x+\frac{4}{3}\pi r$	
ii.	If sum of the surface areas of the box and ball are given to be constant k^2, then x is equal to (a) $\sqrt{\frac{k^2-4\pi r^2}{6}}$ (b) $\sqrt{\frac{k^2-4\pi r}{6}}$ (c) $\sqrt{\frac{k^2-4\pi}{6}}$ (d) None of these	1
iii.	The radius of the ball, when S is minimum, is (a) $\sqrt{\frac{k^2}{54+\pi}}$ (b) $\sqrt{\frac{k^2}{54+4\pi}}$ (c) $\sqrt{\frac{k^2}{64+3\pi}}$ (d) $\sqrt{\frac{k^2}{4\pi+3}}$ OR Minimum value of S is (a) $\frac{k^2}{2(3\pi+54)^{2/3}}$ (b) $\frac{k}{(3\pi+54)^{3/2}}$ (c) $\frac{k^3}{3(4\pi+54)^{1/2}}$ (d) None of these	2
Q.38	**Case Study based-3** A shopkeeper sells three types of flower seeds A_1, A_2 and A_3. They are sold as a mixture where the proportions are 4:4:2 respectively. The germination rates of three types of seeds are 45%, 60% and 35%. Calculate the probability	
i.	of a randomly chosen seed to germinate.	2
ii.	that it is of the type A_2, given that a randomly chosen seed does not germinate.	2
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	“अच्छे लोग और अच्छी किताबें तुरंत समझ में नहीं आते, उन्हें पढ़ना पड़ता है।”	