Applied Mathematics (241)
Marking Scheme
Class XII (2023-24)

Section A
(1 Mark each)

Q-1 Option (c) Here X < 0 and Y > 0 , hence -70 mod 13 is 8 ∵ 8 > 0. 1 Mark

Q-2 Option (b) $x \in (-\infty, -2)$ 1 Mark

$\frac{x+1}{x+2} \geq 1 \Longrightarrow \frac{x+1}{x+2} - 1 \geq 0$

$\Longrightarrow \frac{x+1-x-2}{x+2} \geq 0$

$\Longrightarrow \frac{-1}{x+2} \geq 0 \Longrightarrow x+2 < 0 \left[\because \frac{a}{b} > 0 \text{ and } a < 0 \Longrightarrow b < 0\right]$

$\Longrightarrow x < -2$

Q-3 Option (b) $\bar{x}$ *is a* statistic 1 Mark
Q-4 Option (a) 1 Mark
Q-5 Option (a)
Let man's rate upstream = x km/hr
Let man's rate downstream = 2x km/hr
Hence , Man's rate in still water = $\frac{1}{2}(x + 2x) = \frac{3x}{2}$ km/hr
Therefore $\frac{3x}{2} = 6 \Longrightarrow x = 4\ km/hr$
Man's rate downstream = 8 km/hr
Hence rate of stream $\frac{1}{2}(8 - 4) = 2\ km/h$ 1 Mark

Q-6 Option (d)

x_i	Sample Event	P(x_i) = p_i	$x_i p_i$
0	TT	$\frac{1}{4}$	0
1	HT,TH	$\frac{1}{2}$	$\frac{1}{2}$
2	HH	$\frac{1}{4}$	$\frac{1}{2}$

Mathematical Expectation E(X) = $\sum p_i x_i = 1$ 1 Mark

Q-7 Option (c) $3^1 \equiv 3(mod 7) \Longrightarrow 3^2 \equiv 3 \times 3 = 2(mod 7)$
$\Longrightarrow 3^3 = 3 \times 2 = 6 = -1(mod 7)$
$\Longrightarrow (3^3)^{16} = (-1)^{16}(mod 7)$
$\Longrightarrow (3^3)^{16} = 1(mod 7) \Longrightarrow (3^3)^{16} \times 3^2 = 1 \times 3^2(mod 7)$
$\Longrightarrow 3^{50} = 2(mod 7)$ 1 Mark

Q-8 Option (a) i = $\frac{r}{400}$.

$P = \frac{R}{i} \Longrightarrow 24000 = \frac{300\times400}{r} \Longrightarrow r = \frac{120}{24} = 5\%$ 1 Mark

Q-9 Option (b) $\int \frac{logx}{x} dx$

Put log x = t

Differentiating $\frac{1}{x} dx = dt$

Hence, $\int \frac{logx}{x} dx = \int tdt = \frac{t^2}{2} + C = \frac{(logx)^2}{2} + C$ 1 Mark

Q-10 Option (d) 1 Mark

Q-11 Option (b) 1 mark

$D = \frac{C-S}{n} \Longrightarrow D = \frac{30{,}000-4000}{4} = \frac{26000}{4} = 6500$

Hence, the depreciation is Rs. 6500 1 Mark

Q-12. Option (c)

$$r_{eff} = \left[\left(1+\frac{r}{m}\right)^m - 1\right] \times 100$$

$$r_{eff} = [(1.03)^2 - 1] \times 100 = (1.0609 - 1) \times 100 = 6.09\%$$

Q-13 Option (b)

$$CAGR = \left[\left(\frac{EV}{SV}\right)^{\frac{1}{5}} - 1\right] \times 100$$

$$= \left[\left(\frac{32000}{20000}\right)^{\frac{1}{5}} - 1\right] \times 100$$

$$= \left[(1.6)^{\frac{1}{5}} - 1\right] \times 100$$

$= [1.098 - 1] \times 100 = 0.098 \times 100 = 9.8\%.$ 1 Mark

Q-14 Option (c) $x\frac{dy}{dx} + 2y = x^3$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{x^3}{x}$$

$$\frac{dy}{dx} + \frac{2y}{x} = x^2$$

I.F = $e^{\int\frac{2}{x}} = e^{2lnx} = e^{lnx^2} = x^2$ 1 Mark

Q-15 Option (a) 1 Mark

Q-16 Option (a) 1 Mark

3P(X = 2) = 2P(X = 1)

$\Longrightarrow 3\frac{m^2e^{-m}}{2!} = 2\frac{me^{-m}}{1!} \Longrightarrow m = \frac{4}{3}$

Q-17 Option (c) 1 Mark

Q-18 Option (d)

Z = $\frac{x-\mu}{\sigma} \Longrightarrow 5 = \frac{x-12}{4} \Longrightarrow x = 32.$ 1 Mark

Q-19 Option(c)

Assertion : P(x) = 41 + 24x-8x^2

$P'(x) = 24 - 16x$

$P'(x) = 0 \Longrightarrow 24 - 16x = 0 \Longrightarrow x = \frac{24}{16} = \frac{3}{2}$

$P''(x) = -16 < 0 \Longrightarrow x = \frac{3}{2}$ *is a point of maxima*

Max Profit = P = 41+ $24 \times \frac{3}{2} - 8 \times \frac{9}{4} = 41 + 36 - 18 = 59$

Assertion is true but Reason is false , for Maximum $P'(x) = 0$ *and* $P''(x) < 0.$ 1 Mark

Q-20 Option (a) Both A and R are true and R is the correct explanation of A 1 Mark

Section B
(2 Marks each)

Q-21 Let rate of interest be r% per annum, then i = $\frac{r}{200}$

Given R = Rs 1500 and P = Rs 20,000

P = $\frac{R}{i} \Longrightarrow i = \frac{R}{P} = \frac{1500}{20000}$ 1 Mark

$\Longrightarrow \frac{r}{200} = \frac{1500}{20000} \Longrightarrow r = 15\%$ 1 Mark

Q-22 A = $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$A^2 = A.A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$ 1 Mark

$A^2 = pA \Longrightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = p\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$\Longrightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix} \Longrightarrow p = 4$ 1 Mark

OR

$$\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix}$$ ½ Mark

Comparing a = -2 ; b = -b $\Rightarrow 2b = 0 \Rightarrow b = 0$ *and* $c = -3$ 1½ Mark

Hence a+b+c = -2+0-3 = -5. ½ Mark

Q-23 Let 'x' hectares and 'y' hectares of land be allocated to crop A and Crop B

Max Z = 8000x + 9500y. ½ Mark

Subject to $x + y \leq 10$; $2x + y \leq 50$; $x \geq 0$ *and* $y \geq 0$ 1½ mark

Q-24 $\frac{time\ taken\ upstream}{time\ taken\ downstream} = \frac{2}{1}$ 1 Mark

Let speed of boat = 15 km/hr and speed of stream = y km/hr.

Hence $\frac{15+y}{15-y} = \frac{2}{1}$ ½ Mark

$\Rightarrow 15 + y = 30 - 2y.$

$\Rightarrow$ 3y = 15 $\Rightarrow y = 5\ Km/hr$ ½ Mark

OR

When B runs 50 m A runs 40 m ½ Mark

When B runs 1 m , A runs = $\frac{40}{50} = \frac{4}{5}$ ½ Mark

When B runs 1000 m , A runs = $\frac{4}{5} \times 1000$ = 800 m ½ Mark

Hence B beats A by 200 m ½ Mark

Q-25 Define Null hypothesis H_0 alternate hypothesis H_1 as follows:

H_0: μ = 0.50 *mm*
H_1 : $\mu \neq 0.50$ *mm*
Thus a two-tailed test is applied under hypothesis H_0, we have
$t = \frac{\bar{X}-\mu}{s}\sqrt{n-1} = \frac{0.53-0.50}{0.03} \times 3 = 3.$ 1 Mark
Since the calculated value of t i.e. $t_{cal}(=3) > t_{tab}(=2.262)$, the null hypothesis H_0 can be rejected. Hence, we conclude that machine is not working properly. 1 Mark

Section C
(3 Marks each)

Q-26 $\int \frac{x^3}{(x+2)} dx = \left(\int x^2 - 2x + 4 - \frac{8}{x+2}\right) dx$ 2 Mark

$= \frac{x^3}{3} - x^2 + 4x - 8\ln(x+2) + C.$ 1 *Mark*

(where C is an arbitrary constant of integration)

OR

$\int (x^2+1) ln\, x dx$

Integrating by parts

$lnx\left(\frac{x^3}{3} + x\right) - \int \frac{1}{x}\left(\frac{x^3}{3} + x\right) dx.$ 2 mark

$lnx\left(\frac{x^3}{3} + x\right) - \int \left(\frac{x^2}{3} + 1\right) dx$

$lnx\left(\frac{x^3}{3} + x\right) - \left(\frac{x^3}{9} + x\right)$ + C. 1 Mark

Q-27

Toy A. Toy B

$\begin{bmatrix} 7 & 10 \\ 8 & 6 \end{bmatrix}$ Here Row 1 and Row 2 indicate Shopkeeper 1 and Shopkeeper 2

Cost Matrix = $\begin{bmatrix} 50 \\ 75 \end{bmatrix}$ 1 Mark

Amount = $\begin{bmatrix} 7 & 10 \\ 8 & 6 \end{bmatrix}\begin{bmatrix} 50 \\ 75 \end{bmatrix} = \begin{bmatrix} 350+750 \\ 400+450 \end{bmatrix} = \begin{bmatrix} 1100 \\ 850 \end{bmatrix}$

Income of Shopkeeper P is Rs 1100/ and shopkeeper Q is Rs 850/ 2 Marks

Q-28 f(x) = $2x^3 - 9x^2 + 12x - 5$

$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$

$f'(x) = 6(x-1)(x-2)$ 1 Mark

$f'(x) = 0 \Rightarrow x = 1$ *and* $x = 2$ *are the critical points.* ½ Mark

The intervals are $(-\infty, 1)$; $(1,2)$; $(2, \infty)$ ½ Mark

Increasing in $(-\infty, 1) \cup (2, \infty)$ Decreasing in $(1,2)$ 1 Mark

Q-29 Under pure competition

$p_d = p_s$

$\Rightarrow 16 - x^2 = 2x^2 + 4$

$\Rightarrow 3x^2 = 12 \Rightarrow$ x = 2 , -2 ; since x can't be –ve, so x=2 1 Mark

When $x_0 = 2$; $p_0 = 12$ ½ Mark

Hence, Consumer's surplus = $\int_0^2 p_d dx - p_0 x_0$ ½ Mark

= $\int_0^2 (16 - x^2)\, dx - 12 \times 2$

= 16/3 units 1 Mark

OR

$p_d = p_s$

$\Longrightarrow 56 - x^2 = 8 + \frac{x^2}{3}$

$\Longrightarrow \frac{4}{3}x^2 = 48 \Longrightarrow x^2 = 36 \Longrightarrow$ x = 6 , -6 ; since x can't be –ve, so x=6 1 Mark

When $x_0 = 6\,;\; p_0 = 20$ ½ Mark

Hence, Producer's surplus = $p_0 x_0 - \int_0^6 p_s dx$ ½ Mark

= $6 \times 20 - \int_0^6 \left(8 + \frac{X^2}{3}\right) dx$

= 120 – [48+24]

= 48 units 1 Mark

Q-30 Here P = 5,00,000 ; I = 2,00,000; EMI = 12,500

EMI = $\frac{P+I}{n}$ 1½ Mark

12,500 = $\frac{5{,}00{,}000+2{,}00{,}000}{n} \Longrightarrow$ n = $\frac{7{,}00{,}000}{12{,}500}$ = 56 months. 1½ Mark

Q-31 Let Rs. R be set aside biannually for 10 years in order to have Rs. 500,000 after 10 years

Here S = 500,000. ; n = 10 × 2 = 20

i = $\frac{5}{2\times100} = 0.025$ ½ Mark

R = $\frac{iS}{(1+i)^n-1} = \frac{0.025\times500{,}000}{(1.025)^{20}-1} = \frac{12{,}500}{1.6386-1} = 19{,}574.07.$ 2½ Mark

Section D
(5 Marks each)

Q-32 Here m = 0.4

P$(X = r) = \frac{e^{-m}.m^r}{r!} = \frac{e^{-0.4}\times(0.4)^r}{r!}$

$= \frac{0.6703\times(0.4)^r}{r!}$ 1 Mark

In 1000 pages error = 1000 × $\frac{0.6703\times(0.4)^r}{r!}$ ½ Mark

For zero error P$(X = 0) = 1000 \times \frac{e^{-m}.m^0}{0!} = \frac{e^{-0.4}\times(0.4)^0}{0!}$

$= 1000 \times 0.6703 = 670.3$ 1½ Mark

For one error P$(X = 1) = 1000 \times \frac{e^{-m}.m^1}{1!} = \frac{e^{-0.4}\times(0.4)^1}{1!}$

$= 670.3 \times 0.4 = 268.12$ 2 Mark

OR

Here p = $\frac{1}{2}$ $and\ q = \frac{1}{2}$

P(X=r) = C(n, r)$p^r q^{n-r}$

1-P(r = 0) > $\frac{90}{100}$ 1 Mark

1- C(n,0)$\left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^n > \frac{9}{10}$

$\Rightarrow \quad \frac{n!}{0!(n-0)!}\left(\frac{1}{2}\right)^n < \frac{1}{10}$ 2 Mark

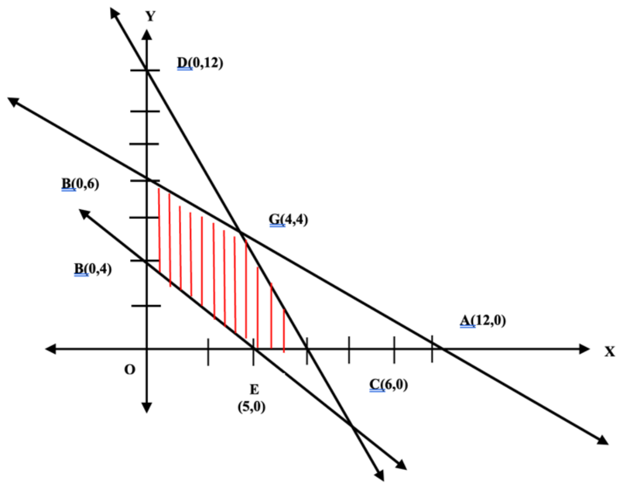
$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{10} \quad \Rightarrow 2^n > 10 \quad \Rightarrow n \text{ is } 4 \text{ or more times}$ 2 Mark

Q-33 Let 'x' and 'y' be the number of units of items M and N respectively.

We have : $x \geq 0 , y \geq 0$

$x + 2y \leq 12 ; 2x + y \leq 12 ; \; x + \frac{5}{4}y \geq 5.$ 1½ Mark

Max Z = 600x + 400 y 1 Mark



Graph 1½ Mark

Corner Point	Z = 600x + 400y
E : (5,0)	3000
C : (6,0)	3600
G : (4,4)	**4000 (Maximum)**
B : (0,6)	2400
F : (0,4)	1600

Hence maximum profit is Rs 4000 when 4 units of each of the items M and N are produced.
1 Mark

Q-34. Let 'x' units of product be produced and sold. As selling price of one unit is Rs 8 total revenue on 'x' units = Rs 8x

(i) Cost Function C(x) = Fixed Cost + 25% of 8x

$= 24000 + \frac{25}{100} \times 8x$

= 24000 +2x. 1 ½ Mark

(ii) Revenue Function = 8x 1 Mark

(iii) Breakeven Point 8x = 24000+2x

x = 4000 1½ Mark

(iv) Profit function = R(x) – C(x) = 6x – 24000 1 Mark

OR

Let x and y be the dimension of the printed pages then x.y = 180.

A = Area of the page = (x+4)(y+5)

= xy +5x +4y + 20

$= 180 + 5x + 4\times\left(\frac{180}{x}\right) + 20$

$= 200 + 5x + \frac{720}{x}$ 2½ Mark

For most economical dimension $\frac{dA}{dx} = 0 \Rightarrow 5 - \frac{720}{x^2} = 0.$

$\Rightarrow x^2 = 144 \Rightarrow x = 12$

Now $\frac{d^2A}{dx^2} = \frac{1440}{x^3}$

$\left(\frac{d^2A}{dx^2}\right)_{x=12} = \frac{1440}{12^3} > 0. \therefore A \text{ is minimum}$

Hence, the most economical dimensions are 16cm and 20 cm 2½ Mark

Q-35 x + y + z = 12

2x + 3y + 3z = 33

x – 2y + z = 0

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$ 1 Mark

$|A| = 3 \neq 0$ ½ Mark

$$\text{adj}A = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$ 2½ Mark

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 108 - 99 + 0 \\ 12 + 0 - 0 \\ -84 + 99 + 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Hence x = 3 , y = 4, z = 5 1 Mark

Section E
(Case Studies Based Questions)

Q- 36 **Case Study – I**

(i)

A + B fill the tank in 6 hrs

B + C fill the tank in 10 hrs

A + C fill the tank in $\frac{15}{2}$ hrs

$2(A + B + C) = \frac{6\times10\times\frac{15}{2}}{6\times10+6\times\frac{15}{2}+10\times\frac{15}{2}} = \frac{450}{60+45+75} = \frac{450}{180} = \frac{5}{2}\ hrs$

Hence A,B and C together will fill the tank in 5 Hrs 2 Mark

(ii) A will in [(A+B+C) – (B+C)] = $\frac{10\times5}{10-5} = 10\ hrs$ 1 Mark

(iii) B will fill in $\frac{\frac{15}{2}\times5}{\frac{15}{2}-5} = 15\ hrs$ 1 Mark

OR

C will fill in $\frac{5\times6}{6-5} = 30\ hrs$

Q-37 **Case Study – II**

x_i	0	1	2	3	4	5
P(X=x_i)	0.2	k	2k	2k	0	0

(i) Since $\sum P = 1 \implies 0.2 + k + 2k + 2k = 1 \implies 0.2 + 5k = 1 \implies 5k = --0.2$

$\implies$ k = $\frac{4}{25}$ 1½ Mark

(ii) P(X=2) = 2k = $\frac{8}{25}$ 1 Mark

(iii) P(X $\geq$ 2) $= 4k = \frac{16}{25}$ 1½ Mark

OR

P($X \leq 2$) = 0.2+3k $= \frac{17}{25}$

Q-38 **Case Study – III**

Year	Y	X=Year - 2003	X^2	XY
2001	160	-2	4	-320
2002	185	-1	1	-185
2003	220	0	0	0
2004	300	1	1	300
2005	510	2	4	1020
	1375		10	815

2 Marks for table

a = $\frac{\sum Y}{n}$ = $\frac{1375}{5}$ = 275 ½ Mark

b = $\frac{\sum XY}{\sum X^2} = \frac{815}{10}$ = 81.5 ½ Mark

Y_c = a + bX

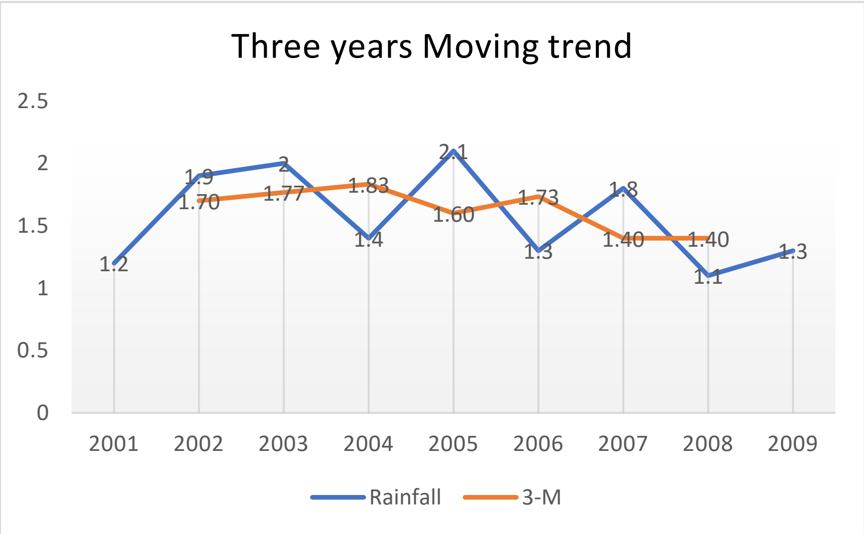
Y_c = 275 + 81.5 X

The estimated value for 2008 will be 275 + 151.5 × $5 = 275 + 757.5 = 1032.5$. 1 Mark

OR

Year	Rainfall(in cm)	3 years moving total	3 years moving average
2001	1.2		
2002	1.9	5.1	1.70
2003	2	5.3	1.77
2004	1.4	5.5	1.83
2005	2.1	4.8	1.60
2006	1.3	5.2	1.73
2007	1.8	4.2	1.40
2008	1.1	4.2	1.40
2009	1.3		

1½ Marks for table



2½ Mark Marks for graph