Maple Online Classes

A-25 DLF Loni Ghaziabad UP 201301

TEST PAPER: DERIVATIVE AND APPLICATION

Class 12 - Mathematics

Time Allowed: 3 hours and 20 minutes **Maximum Marks: 100**

General Instructions:

All the questions are compulsory.

- There are 4 Sections of Questions as A, B, C and D.
- Section A has 10 Questions of 1 mark each.
- Section B has 15 Questions of 2 marks each.
- Section C has 10 Questions of 3 marks each.
- Section D has 5 Questions of 6 marks.
- There are total 40 Questions in the Question paper.
- ***Note: You have given 20 minutes extra to read and understand the questions in questions paper. After the 20 minutes you are allowed to start writing answer on the answer sheet.***

Section A

1. Differentiate the function with respect to x : $\frac{\sin(ax+b)}{\cos(cx+d)}$ **[1]**

2. Differentiate the function with respect to x : $\cos x^3 \sin^2\left(x^5\right)$ **[1]**

3. Prove that the function f given by f(x) = |x - 1|, x $\in$ R is not differentiable at x = 1. **[1]**

4. Find $\frac{dy}{dx}$ if ax + by^2 = cos y **[1]**

5. Find $\frac{dy}{dx}$ if sin^2 y + cos xy = κ **[1]**

6. Differentiate $e^{sin^{-1}x}$ w.r.t. x. **[1]**

7. Differentiate the function $(\log x)^{\cos x}$ w.r.t. x. **[1]**

8. If x and y are connected parametrically by the equation x = 4t, $y = \frac{4}{t}$, without eliminating the parameter, find $\frac{dy}{dx}$. **[1]**

9. Find the second-order derivative of the function e^{x}sin 5x **[1]**

10. Find the second-order derivative of the function log(log x) **[1]**

Section B

11. A stone is dropped into a quiet lake and waves move in circles at a speed of 4 cm/s. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing? **[2]**

12. Show that the function given by f(x) = 7x – 3 is increasing on R. **[2]**

13. Find the intervals in which the function f given by f(x) = x^2 – 4x + 6 is **[2]**

    a. increasing

    b. decreasing

14. Find the maximum and the minimum values, if any, of the function f given by $f(x) = x^2$, $x \in R$. **[2]**

15. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5$. **[2]**

16. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm. **[2]**

17. Show that the function given by f (x) = sin x is decreasing in $\left(\frac{\pi}{2}, \pi\right)$ **[2]**

18. Let I be any interval disjoint from [-1, 1]. Prove that the function f given by $f(x) = x + \frac{1}{x}$ is increasing on I. **[2]**

19. Find the maximum and minimum value, f(x) = |x + 2| - 1 **[2]**

20. Find the local maxima and local minima of the function **[2]**
$f(x) = x^3 - 6x^2 + 9x + 15$
Find also the local maximum and the local minimum value.

21. Prove that the function does not have maxima or minima: g(x) = log x **[2]**

22. Find the intervals in which the function f given by $f(x) = \frac{4 \sin x - 2x - x \cos x}{2 + \cos x}$ is **[2]**
 i. increasing
 ii. decreasing

23. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at x = e. **[2]**

24. Find the absolute maximum and minimum values of the function f given by f(x) = $\cos^2 x$ + sin x, x $\in$ [0, π] **[2]**

25. Let f be a function defined on [a, b] such that f ′(x) > 0, for all x $\in$ (a, b). Then prove that f is an increasing function on (a, b). **[2]**

Section C

26. Differentiate the function $x^x - 2^{\sin x}$ w.r.t. x. **[3]**

27. Find the derivative of the function given by f(x) = (1 + x) (1 + x^2) (1 + x^4) (1 + x^8) and hence find f ′(1). **[3]**

28. If u, v and w are functions of x, then show that **[3]**
$\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w + u.\frac{dv}{dx} \cdot w + u \cdot v\frac{dw}{dx}$
in two ways - first by repeated application of product rule, second by logarithmic differentiation.

29. If y = Ae^{mx} + Be^{nx}, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$ **[3]**

30. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long? **[3]**

31. Prove that the function f given by f (x) = log |cos x| is decreasing on $\left(0, \frac{\pi}{2}\right)$ and increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$. **[3]**

32. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. **[3]**

33. A man of height 2 metres walks at a uniform speed of 5 km/h away from a lamp post which is 6 metres high. Find the rate at which the length of his shadow increases. **[3]**

34. If y = 3 cos (log x) + 4 sin (log x). Show that $x^2y_2 + xy_1 + y = 0$ **[3]**

35. If x and y are connected parametrically by the equation $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, y = a sin t, without eliminating the parameter, find $\frac{dy}{dx}$. **[3]**

Section D

36. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening. **[6]**

37. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3\tan^2\alpha$. **[6]**

38. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is$(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$ **[6]**

39. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs Rs 70 per sq.metres for the base and Rs 45 per sq. metre for sides. What is the cost of least expensive tank ? **[6]**

40. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area. **[6]**