CODE:0402- AG-6-23-24 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

EXAMINATION 2023 -24

Time : 3 Hours | Maximum Marks : 80

CLASS – XII | **MATHEMATICS**

Sr. No.	**SECTION – A** This section comprises of very short answer type-questions (VSA) of 2 marks each	Marks
Q.1	If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$ and $A \ adj \ A=\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then k is equal to (a) 0 (b) 1 (c) $\sin\alpha\cos\alpha$ (d) $\cos 2\alpha$	1
Q.2	Assume X, Y, Z, W and P are matrices of order 2 × n, 3 × k, 2 × p, n × 3 and p × k, respectively . The restriction on n, k and p so that PY + WY will be defined are: (A) k = 3, p = n (B) k is arbitrary, p = 2 (C) p is arbitrary, k = 3 (D) k = 2, p = 3	1
Q.3	If $D=\begin{vmatrix}1 & 2 & 3\\ 2 & -1 & 0\\ 3 & 4 & 5\end{vmatrix}$ then $\begin{vmatrix}1 & 6 & 3\\ 4 & -6 & 0\\ 3 & 12 & 5\end{vmatrix}$ is equal to a . 6D b. 3D c. 0 d. 2D	1

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Q.4	If $f(x)=\begin{cases} x\sin x, \text{ when } 0<x\le\frac{\pi}{2} \\ \frac{\pi}{2}\sin(\pi+x), \text{ when } \frac{\pi}{2}<x<\pi \end{cases}$, then (a) $f(x)$ is discontinuous at $x=\pi/2$ (b) $f(x)$ is continuous at $x=\pi/2$ (c) $f(x)$ is continuous at $x=0$ (d)None of these	1
Q.5	If θ be the angle between the vectors a and b and $\lvert\vec{a}\times\vec{b}\rvert=\vec{a}\bullet\vec{b}$, then $\theta=$ (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) o	1
Q.6	An integrating factor of the differential equation $(1-x^2)\frac{dy}{dx}-xy=1$, is (a) $-x$ (b) $-\frac{x}{(1-x^2)}$ (c) $\sqrt{(1-x^2)}$ (d) $\frac{1}{2}\log(1-x^2)$	1
Q.7	For an objective function $Z=ax+by$, where $a, b>0$; the corner points of the feasible region determined by a set of constraints (linear inequalities) are (0, 20), (10, 10), (30, 30) and (0, 40). The condition on a and b such that the maximum Z occurs at both the points (30, 30) and (0, 40) is: a) $b-3a=0$ b) $a=3b$ c) $a+2b=0$ d) $2a-b=0$	1
Q.8	If a line makes angles $\alpha,\beta,\gamma,\delta$ with four diagonals of a cube, then the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta$ is (a) $\frac{4}{3}$ (b) 1(c) $\frac{8}{3}$ (d) $\frac{7}{3}$	1
Q.9	The value of $\int_{-2}^{3}\lvert 1-x^2\rvert\,dx=$ (a) $\frac{1}{3}$ (b)$\frac{14}{3}$(c)$\frac{7}{3}$(d) $\frac{28}{3}$	1
Q.10	The roots of the equation $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}=0$ are (a) 0, – 3 (b) 0, 0, – 3 (c)0, 0, 0, – 3 (d) None of these	1
Q.11	In a factory which produces two products A and B, in manufacturing product A, the machine and the carpenter requires 3 hours each and in manufacturing product B, the machine and carpenter requires 5 hour and 3 hour respectively. The machine and carpenter works at most 80 hour and 50 hour per week respectively. The profit on A and B are Rs. 6 and Rs. 8 respectively. If profit is maximum by manufacturing x and y units of A and B type products respectively, then for the function 6 x + 8y , the constraints are (a) $x\ge 0, y\ge 0, 5x+3y\le 80, 3x+2y\le 50$ (b) $x\ge 0, y\ge 0, 3x+5y\le 80, 3x+3y\le 50$	1

	(c) $x \geq 0, y \geq 0, 3x + 5y \geq 80, 2x + 3y \geq 50$ (d) $x \geq 0, y \geq 0, 5x + 3y \geq 80, 3x + 2y \geq 50$	
Q.12	If the position vectors of three points A, B and C are respectively i + j + k, 2i + 3j – 4k and 7i + 4j + 9k, then the unit vector to the plane containing the triangle ABC is (a) 31i – 18j – 9k(b) $\frac{31\mathbf{i} - 38\mathbf{j} - 9\mathbf{k}}{\sqrt{2486}}$ (c) $\frac{31\mathbf{i} + 18\mathbf{j} + 9\mathbf{k}}{\sqrt{2486}}$ (d) None of these	1
Q.13	If I_3 is the identity matrix of order 3, then Cofactor of I is (a) 0 (b) $3I_3$ (c) I_3 (d) Does not exist	1
Q.14	Probability that A speaks truth is 4/5 . A coin is tossed. A reports that a head appears. The probability that actually there was head is A. 4/5 B.1/2 C.1/5 D.2/5	1
Q.15	The solution of the equation $\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$ is (a) $\log\left(\frac{y}{x}\right) = cx$ (b) $\frac{y}{x} = \log y + c$ (c) $y = \log y + 1$ (d) $y = xy + c$	1
Q.16	The area of the triangle having vertices as $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $-2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, $4\mathbf{i} - 7\mathbf{j} + 7\mathbf{k}$ is (a) 26(b)11(c) 36 (d) 0	1
Q.17	If A=R – {3}, B=R–{1}. Let $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall x \in A.$, then find x if $f^{-1}(x) = 4$ (a) *2* (b) - 2 (c) Does not exist because *f* is not bijective (d) none of these	1
Q.18	The length of the perpendicular drawn from the point (5, 4, –1) on the line $\frac{x-1}{2} = \frac{y}{9} = \frac{z}{5}$ is (a) $\sqrt{\frac{110}{2109}}$ (b) $\sqrt{\frac{2109}{110}}$ (c) $\frac{2109}{110}$ (d) 54 (b) $\sqrt{\frac{2109}{110}}$	1
	ASSERTION-REASON BASED QUESTIONS In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
Q.19	Assertion (A): The function f(x) tanx – x always increases . Reason (R): The value(s) of x for which $f'(x) > 0$, $f(x)$ is increasing; and the value(s) of x for which $f'(x) < 0$, $f(x)$ is decreasing.	1

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Q.20	**Assertion (A) :** The function $f(x)=\sin\left(\log(x+\sqrt{x^2+1})\right)$ is even function . **Reason (R) :** (i) f(-x) = f(x) therefore f(x) is even function . (i) f(-x) = - f(x) therefore f(x) is odd function .	1
	SECTION – B **This section comprises of very short answer type-questions (VSA) of 2 marks each**	
Q.21	Find all point of discontinuity of f , where f is defined as following : $f(x)=\begin{cases} \|x\|+3 & if x\le -3 \\ -2x & -3<x<3 \\ 6x+2 & if x\ge 3 \end{cases}$.	2
Q.22	Prove that : $\sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)=\sqrt{1-x^2}$ **OR** Reduced in simplest form $\tan^{-1}\left(\frac{7ax}{a^2-12x^2}\right)$.	2
Q.23	The two vector $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two side vectors $\vec{AB}$ and $\vec{AC}$ respectively of triangle ABC. Find the length of the median through A.	2
Q.24	Find the interval in which $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is (a) strictly increasing (b) strictly decreasing **OR** If the function $f(x)=2x^3-9ax^2+12a^2x+1,$ where $a>0$ attains its maximum and minimum at p and q respectively such that $p^2=q$, then find the value of a .	2
Q.25	Determine the intervals in which the function f given by $f(x)=\log(1+x)-\frac{2x}{2+x}, x\neq -2$ Is increasing or decreasing . OR Find the interval in which $f(x)=\frac{4\sin x-2x-x\cos x}{2+\cos x}$ on $(0,2\pi)$ is (a) increasing (b) decreasing.	2
	SECTION – C **(This section comprises of short answer type questions (SA) of 3 marks each)**	
Q.26	Find: $\int\frac{e^x(x-3)}{(x-1)^3}dx$.	3
Q.27	Evaluate $\int\frac{x\sin^{-1}(x^2)}{\sqrt{1-x^4}}dx$	3

	OR Evaluate: $\int_0^{\pi/4}\log(1+\tan\theta)\,d\theta$.	
Q.28	A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. OR If A and B are two events such that $2P(A)=P(B)=5/13\,\&\,P(A/B)=2/5$, find $P(A\cup B)$.	3
Q.29	Solve $\frac{dy}{dx}+y\sec^2 x=\tan x\sec^2 x$. OR The solution of the differential equation $(3xy+y^2)dx+(x^2+xy)dy=0$.	3
Q.30	If $x^m y^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.	3
Q.31	There is a factory located at each of the two places P & Q . From these locations , a certain commodity is delivered to each of the three depots situated at A , B & C. The weekly requirements of the depots 5 , 5 & 4 units of commodity while the production capacity of the factories at P & Q are respectively 8 & 6 units .The cost of transportation per unit is given below. Formulate the above L.P.P. mathematically to determine how many units should be transported from each factory to each depot in order that the transportation cost is minimum.	3

TO	COST (in Rs)		
	A	B	C
P	16	10	15
Q	10	12	10

	SECTION – D **(This section comprises of long answer-type questions (LA) of 5 marks each)**	
Q.32	Show that the lines $\frac{x-1}{3}=\frac{y-1}{-1}, z+1=0$ and $\frac{x-4}{2}=\frac{z+1}{3}, y=0$ intersect each other. Also find their point of intersection. **OR** Prove that the lines whose direction cosines are given by al + bm + cn = 0, fmn + gnl + hlm = 0 are (I)perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$ (II)parallel if $a^2f^2+b^2g^2+c^2h^2-2(bcgh+cahf+abfg)=0$.	5
Q.33	Using integration find the area of the region bounded by the circle $x^2+y^2=32$, x axis and the line $x=y$ in the first quadrant .	5
Q.34	Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): $\lvert a-b\rvert$ is even} is an equivalence relation. Show that all the elements of {1, 3, 5} are related to	5

	each other and the elements of {2, 4} are related to each other. But no elements of {1, 3, 5} is related to any elements of {2, 4} . **OR** Show that the function $f : N \to N$, given by f(x) = $x - (-1)^x$, is a bijection	
Q.35	If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equation :x – 2y = 10 , 2x + y + 3z = 8 & -2y + z = 7 .	5
	SECTION – E (This section comprises of 3 case study / passage – based questions of 4 marks each with two sub parts (i),(ii),(iii) of marks 1, 1, 2 respectively.The third case study question has two sub – parts of 2 marks each.)	
Q.36	**Case Study based-1** A & B appear for an interview for two vacancies in the same post . The probability of A selection is $\frac{1}{6}$ and B is $\frac{1}{4}$. Assuming the two events as independent find the probability of	
i.	Both students passing the examination.	1
ii.	Only one of them is selected .	1
iii.	at least one of them is selected . OR none of them is selected	2
Q.37	**Case Study based-3** Shobhit's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in figure. He has 200 ft of wire fencing.   Based on the above information, answer the following questions.	
i.	If x denote the length of the side of the garden perpendicular to the brick wall and y denote the length of the side parallel to the brick wall, then find the relation representing total amount of fencing wire. (a) $x + 2y = 150$ (b) $x + 2y = 50$ (c) $y + 2x = 200$ (d) $y + 2x = 100$	1

ii.	Maximum are of garden will be (a) 2500 sq. ft (b) 4000 sq. ft (c) 5000 sq. ft (d) 6000 sq. ft	1
iii.	Area of the garden as a function of x, say $A(x)$, can br represented as (a) $200 + 2x^2$ (b) $x - 2x^2$ (c) $200x - 2x^2$ (d) $200 - x^2$ **OR** Maximum value of $A(x)$ occurs at x equals (a) 50 ft (b) 30 ft (c) 26 ft (d) 31 ft	2
Q.38	**CASE STUDY- 3** Two skew lines $\frac{x-6}{3} = -(y-7) = (z-4)$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$. Hence find the shortest distance between the given lines.	
i.	Find the points on the line which are nearest to each other .	2
ii.	Find the shortest distance between the given lines.	2
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	“मेहनत करो, सफलता खुद आपके पास आएगी।”	